Math 375 Review Exam 1

Below I list the main topics we covered so far, questions you should know how to answer, and specific skills you should have for the exam. Good related problems are those in HW 2 through HW 6.

- **Taylor series**

  - Write Taylor polynomial approximations $p_N(h)$ of $f(a+h)$ as functions of h, for small h, and state a formula for the error $f(a+h) - p_N(h)$.

  - An approximation $g(h)$ of $f(h)$ is of order p if $g - f = O(h^p)$. Determine the order p of an approximation by plotting $|g - f|$ on a log-log scale.

  - Given plots of $|g - f|$ on a log-log scale, explain the slopes you observe, referring to Taylor series if needed.

  - Use Taylor series to prove convergence and derive the order of iterative schemes (fixed point iterations, Newton's method).

  - Use Taylor series to derive the order of a finite difference approximation of a derivative.

- **Numerical methods**

  - What methods did we discuss to solve $f(x) = 0$? For each method
    * give a formula for the solution at the kth step
    * give a picture of the sequence of solutions, if possible (such as for FPI, and Newton's)
    * what is the order of the method, under which conditions?
    * given the error at the kth step, can you approximate the error at the $k+1$st step?

  - Estimate the number of steps Bisection Method requires to obtain the solution to within p digits of precision.

  - What methods did we discuss to solve $Ax = b$, or triangular systems? Know related operation counts, and accuracy.

  - Find LU and PLU factorizations of matrices. How would you use these to solve $A\mathbf{x}_k = \mathbf{b}_k$ for many right hand sides $k = 1, \ldots$

  - Use Gauss Elimination and backward substitution to solve $Ax = b$.

- **Algorithms and Matlab**

  - write down algorithms that implement the methods we discusseed to solve a problem (solve nonlinear equations, solve linear systems)

  - write down matlab code for those algorithms

  - obtain operation counts for simple algorithms

- use operation counts to estimate the time it takes to execute a given code on a machine for which a sample time is known.

- State results of a given matlab code.

- **Accuracy**

  - What is machine precision?

  - How can accuracy be lost in a numerical simulation? Give specific examples.

  - What is the forward error when solving $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$? What is the relative forward error? What is the backward and relative backward error? How are foward and backward errors related?

  - What is the condition number of a matrix? What does it measure? Find condition numbers for 2x2 matrices.

  - What is an ill-conditioned problem?

  - What is the order of convergence of an iterative method? If the method is of order p, by how much does the error decay at each step?

  - If an approximation error is $O(h^p)$, by how much does the error decay each time h is halved?