(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

$y = x^3 - 2x^2 + x + 8$

$y' = 3x^2 - 4x + 1$

$y'' = 6x - 4$

slope goes from + to − or − to +

$6x - 4 = 0$

$6x = 4 \quad x = \frac{2}{3}$

$y(\frac{2}{3}) = 3(\frac{-8}{27}) - 2(\frac{4}{9}) + \frac{2}{3} + 8 = \frac{-8}{27} - \frac{24}{27} + \frac{18}{27} + \frac{216}{27} = \boxed{\frac{202}{27}}$

$(\frac{2}{3}, \frac{202}{27})$

$y'(\frac{2}{3}) = 3(\frac{4}{9}) - 4(\frac{2}{3}) + 8$

$= \frac{4}{3} - \frac{8}{3} + \frac{24}{3} = -\frac{4}{3} + \frac{24}{3} = \boxed{\frac{20}{3}}$

(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

$$y'(x) = 3x^2 - 4x + 1$$

$$y''(x) = 6x - 4$$

$$y''(x) = 6x - 4 = 0 \quad \text{when} \quad 6x - 4 = 0$$

$$6x = 4$$

$$x = \frac{2}{3}$$

$$y'\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 1$$

$$= 3\left(\frac{4}{9}\right) - \frac{8}{3} + 1 = \frac{12}{9} - \frac{24}{9} + 1 = \frac{-3}{9} = -\frac{1}{3}$$

(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

OPTIMIZATION! MINIMUM VALUE!

$y = x^3 - 2x^2 + x + 8$

$y' = 3x^2 - 4x + 1$

$y' = 0$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$= 3x^2 - 3x - x + 1$

$= 3x^2 - 4x + 1$ ✓

CRITICAL POINTS:

$x = \frac{1}{3}$ and $x = 1$

closed interval

first derivative test

max

$+ \quad - \quad +$

$\frac{1}{3} \qquad 1$

MIN

second derivative Test:

$y'' = 6x - 4$

$y''\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right) - 4 = \frac{6}{3} - 4 = 2 - 4 = -2 < 0$ CD MAX

$y''(1) = 6(1) - 4 = 6 - 4 = 2 > 0$ CU MIN

$y(1) = 1^3 - 2(1) + 1 + 8$

$= 1 - 2 + 1 + 8$

$= 8$

$= (1, 8)$ ← coordinates of minimum

$y\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} + 8$

$= \frac{1}{27} - 2\left(\frac{1}{9}\right) + \frac{1}{3} + 8$

$= \frac{1}{27} - \frac{2}{9} + \frac{25}{3}$

$= \frac{1}{27} - \frac{6}{27} + \frac{225}{27} = \frac{220}{27}$

$\left(\frac{1}{3}, \frac{220}{27}\right)$

$m = \dfrac{\frac{216}{27} - \frac{220}{27}}{\frac{3}{3} - \frac{1}{3}} = \boxed{-2/9}$ ← smallest slope

$\boxed{y = -\frac{2}{9}x + \frac{80}{9}}$