

(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

$$y = x^3 - 2x^2 + x + 8$$

$$y' = 3x^2 - 4x + 1$$

$$y'' = 6x - 4$$

slope goes from + to - or - to +

$$6x - 4 = 0$$

$$6x = 4 \quad x = \frac{2}{3}$$

$$y\left(\frac{2}{3}\right) = 3\left(\frac{-8}{27}\right) - 2\left(\frac{4}{9}\right) + \frac{2}{3} + 8 = \frac{-8}{27} - \frac{24}{27} + \frac{18}{27} + \frac{216}{27} = \boxed{\frac{202}{27}}$$

$$\left(\frac{2}{3}, \frac{202}{27}\right)$$

$$y'\left(\frac{2}{3}\right) = 3\left(\frac{4}{9}\right) - 4\left(\frac{2}{3}\right) + 1$$

$$= \frac{4}{3} - \frac{8}{3} + \frac{24}{3} = \frac{-4}{3} + \frac{24}{3} = \boxed{\frac{20}{3}}$$

(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

$$y'(x) = 3x^2 - 4x + 1$$

$$y''(x) = 6x - 4$$

$$y''(x) = 6x - 4 = 0 \quad \text{when} \quad 6x - 4 = 0$$

$$6x = 4$$

$$x = \left(\frac{2}{3}\right)$$

$$y'\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 1$$

$$= 3\left(\frac{4}{9}\right) - \frac{8}{3} + 1 = \frac{12}{9} - \frac{24}{9} + 1 = \frac{-3}{9} = \left(-\frac{1}{3}\right)$$

(10 points) Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. What is the slope at that point?

OPTIMIZATION! MINIMUM VALUE!

$$y = x^3 - 2x^2 + x + 8$$

$$y' = 3x^2 - 4x + 1$$

$$y' = 0$$

$$3x^2 - 4x + 1 = 0$$

$$(3x-1)(x-1) = 0$$

$$= 3x^2 - 3x - x + 1$$

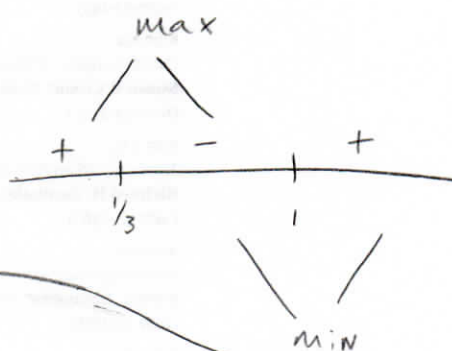
$$= 3x^2 - 4x + 1 \quad \checkmark$$

CRITICAL POINTS:

$$x = \frac{1}{3} \text{ and } x = 1$$

closed interval

first derivative test



second derivative Test:

$$y'' = 6x - 4$$

$$y''\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right) - 4 = \frac{6}{3} - 4 = 2 - 4 = -2 < 0 \quad \text{CD} \quad \text{MAX} \quad \curvearrowright$$

$$y''(1) = 6(1) - 4 = 6 - 4 = 2 > 0 \quad \text{CU} \quad \text{MIN} \quad \cup$$

$$y(1) = 1^3 - 2(1) + 1 + 8$$

$$= 1 - 2 + 1 + 8$$

$$= 8$$

$$= (1, 8) \quad \leftarrow \text{coordinates of minimum}$$

$$y\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} + 8$$

$$= \frac{1}{27} - 2\left(\frac{1}{9}\right) + \frac{1}{3} + 8$$

$$= \frac{1}{27} - \frac{2}{9} + \frac{25}{3}$$

$$= \frac{1}{27} - \frac{6}{27} + \frac{225}{27} = \frac{220}{27}$$

$$\left(\frac{1}{3}, \frac{220}{27}\right)$$

$$m = \frac{\frac{216}{27} - \frac{220}{27}}{\frac{2}{3} - \frac{1}{3}} = \boxed{-\frac{2}{9}} \quad \leftarrow \text{smallest slope}$$

$$\boxed{y = -\frac{2}{9}x + \frac{80}{9}}$$