Senior Thesis Proposal

Student: Alex Benedict

Advisor: Stephen Lau

There is a need for accurate modeling of gravitational waves. Currently there are a large number of projects designed to detect gravitational waves (the ground based detectors LIGO, VIRGO, GEO600, TAMA), and space-based detectors (e.g. LISA). Yet, gravitational waves are expected to be very weak and difficult to detect. Therefore, accurate modeling of expected waveforms is important. The Schwarzschild solution is a spherically symmetric vacuum solution to the Einstein equations. Perturbations of the Schwarzschild solution can be described in terms of a denumerable collection of 1+1 "master" wave equations (Regge-Wheeler and Zerilli equations, indexed by spherical harmonic indices):

$$-\partial_{tt}\psi + \partial_{xx}\psi - V(r)\psi = S(r,t)$$

where r is the areal radius from the center of the hole, t is time, V(r) is a potential, S(r,t) is a source term, and x = r + 2M log(r/2M - 1) the Regge-Wheeler tortoise coordinate. The difference between the Regge-Wheeler and Zerilli equations is in the potential, they describe perturbations of differing parity. These wave equations represent a simplified model of the situation and are accurate for small perturbations of non-rotating, non-charged, spherically symmetric black holes. They can accurately model the case where an orbiting "particle" has a mass much much less than that of the hole, a so-called Extreme Mass Ratio Binary (EMRB). Time-domain simulations for such scenarios require inner and outer boundary conditions at the inner and outer edges of the computational domain. Boundary Conditions (BC's) for the inner radius are trivial since V(r) decays exponentially fast for $x \to -\infty$ (or in physical space as $r \to 2M$, the gravitational radius). Typically, time-domain simulations adopt only approximate outgoing radiation boundary conditions and therefore require a correspondingly large outer radius since the potential decays approximately as the inverse square of the radius. The advantage of using exact nonlocal radiation boundary condition (RBC) instead of approximate RBC is that it allows for smaller computational domains. The exact RBC involves temporal convolutions of a kernel (which can be tabulated ahead of time) with the solution psi on the boundary.

Related to this, is the need for the asymptotic waveform. Since most sources of gravitational waves are very far away (>20 light years), one would like to know what the expected waveforms look like without actually running the simulation for correspondingly large times. In a similar manner to the RBC kernels one can derive an extraction kernel, which can "extract" the solution at a larger radius through a convolution of the solution psi on the boundary with the kernel. The project would involve the extended precision computation and testing of radiation and extraction kernels for black hole perturbations. As part of the project, I may also investigate the wellposedness of the RBC.

The RBC kernels have already been computed in [1], which used double precision accuracy to compute kernels satisfying error tolerances below double precision; however, there is a need for kernels, which satisfy double precision tolerances. The task of computing such compressed (i.e. numerical) kernels is ideal for extended precision, since it only needs to be done once, and only a small number of very accurate terms are needed to establish accurate results. Having more accurate compressed kernels for the Regge-Wheeler and Zerilli, equations would help lower error in certain numerical simulations of gravitational waves without adding extra computational costs. Similarly, having extended precision tables for the extraction kernels would allow for accurate approximation of the asymptotic waveforms, which is what detectors would actually be looking for, given the large distance between the sources and detectors. Proving the wellposedness of the RBC is important theoretically and would suggest the procedure computationally stable. Empirical evidence suggests it is indeed stable, but as of yet there is no formal proof.

[1]S. R. Lau, "Analytic structure of radiation boundary kernels for blackhole perturbations," J. Math. Phys. 46, article id. 102503 (21 pages), 2005.