Honors Thesis Proposal
Population Dynamics

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Abstract

The goal of this project is to model a species' population growth and movement over a finite region. This will be accomplished by combining predator-prey equations governing population dynamics with probabilistic movement equations, both of which are based on a grid of resource conditions.

1 Conceptual Overview

A standard predator-prey model of population accounts for birth rate, death rate, predation, and environmental carrying capacity. While often useful for determining temporal population dynamics, this model contains no spatial information. The goal of this research project is to find a novel way to model both population growth and spread.

Animals tend to move to areas with better resource availability and more favorable terrain. As these animals consume resources, breed, and deal with changing weather conditions, the attractiveness of the area they inhabit will change. This change inevitably leads to the animals moving again, which is one mechanism for their spread. To account for these factors, we propose a two-step model. First, a surface that represents the how hospitable an animal's surroundings are will be created. This surface will be based on a set of predator-prey equations, GIS data, and seasonal considerations. Then, once the surface is in place, it will be used to determine how the animals will move. As the animals move and consume resources locally, the surface must be updated to reflect these changes.

1.1 Proposed Methodology

The first task will be to create an environment the model can run in. To accomplish this, a suitable topographical map will be downloaded from a USGS data bank. This map will contain information on rivers, mountains and other geographical features which will allow for the quantification of passability conditions. A grid will be overlaid on the map in order to break it into small pieces and provide a baseline of terrain-passability for each square. The terrain passability will be a scalar number, τ, that factors in relative elevation as well as natural obstacles. If, for instance, a mountain or river falls in a particular grid square, the overall terrain-passability score for that square would be very low.

Weather can greatly affect both terrain passability and resource availability. To account for this, seasonal dependence will be factored into the the model. We intend to derive weather conditions from real world data on the modeled region. It is assumed that weather is, in general, similar from year to year. Research will be conducted to find the best way to average weather conditions over some amount of years. This data will then be modeled as a time dependent periodic function.

After developing an environment, another set of conditions will be applied to account for food availability. The species modeled here is predatory, so the food availability will be modeled as prey population. To accomplish this a set of predator-prey equations will be assigned to each grid square:

$$\frac{dP_1}{dt} = r_1(t)(1 - \frac{P_1}{k_1(t)}) + c_1 P_1 P_2 \tag{1a}$$

$$\frac{dP_2}{dt} = r_2(t)(1 - \frac{P_2}{k_2(t)}) - c_2 P_1 P_2 \tag{1b}$$

Where P_1 is the predator population, P_2 is the prey population, $r(t)$ is the seasonally dependent growth rate, $k(t)$ is the seasonally dependent carrying capacity and c is a constant governing how beneficial or detrimental the interaction is.

After these factors are calculated, they will be aggregated together as a coefficient of attractiveness:

$$\alpha_{i,j} = g(\tau_{i,j}(t), P_{1,i,j}(t), P_{2,i,j}(t)) \tag{2}$$

After α_{ij} is calculated for all of the nodes, the data will be interpolated to form a surface of attractiveness using either bi-linear or bi-cubic interpolation to give a continuous surface on which to migrate our species.

Once the surface is in place, the next step will be to evolve the predators' position. We will attempt do this using both Lagrangian and Eularian formulations. In the Lagrangian case, research will be conducted into several possible equation candidates to imbue a representative predator with some sort of velocity. In this formulation, the predator is assumed to have agency and to employ some decision-making process. This process would involve the predator taking in data from the surrounding region and then deciding to move according to this input. At the most basic, a predator will simply move to the area with the most to offer. In this formulation, the predator's velocity would be given by the partial differential equation:

$$\overline{v} = \nabla\alpha(x, y) \tag{3}$$

Additionally, research into probabilistic movement formulations will be conducted:

$$\overline{v} = P(\alpha(x, y), t, Te) \tag{4}$$

where $P(\alpha(x, y), t, Te)$ is a probabilistic equation, $\alpha(x, y)$ is attractiveness at a point, t is time, and Te is a measure of how territorial the animals are.

Research into possible Eularian formulations will be conducted as well. Instead of tracking individual predators, this formulation tracks how the concentration of predators in a region changes. The spread of predators is decided by the attractiveness of the surrounding area, a region's current population, and seasonal behaviors.

Both the predator-prey and movement equations require their solutions to be approximated numerically in any formulation. To accomplish this, Runge-Kutta methods will be employed for the predator-prey equations as well as finite difference schemes to approximate solutions for the partial differential equations. The computing will be done in MatLab.

2 A Brief Timeline

2.1 Progress Thus Far

Work so far has consisted mainly of reading a number of research papers on various differential equation models. Aside from the literature search, basic research has been conducted into fluid modeling concepts, such as Eulerian and Lagrangian formulations, as well as some probability theory.

2.2 Timeline

Initially, research into more advanced ordinary and partial differential equations will be of the most importance. These subjects will take a large part of the first semester, but are fundamental to the model. Alongside this work, research into current population dynamics models is needed. To accomplish this, a more exhaustive literature search will be employed, and will focus not only on population, but also on disease models and various agent-based models.

Once the fundamental differential equations have been researched, we will move into writing the code to employ the model. This side of the project requires study in numerical differential equations and interpolation. Much of this knowledge will come from hands-on coding in MatLab throughout the course of the second semester.