

Abstract

This research project is a collection of several ideas, condensed mainly into three broader concepts. These ideas cover what can be considered as perhaps the milestones of the option pricing theory.

The first idea results from exploring the consequences of modifying an assumption of one of the most important results in financial mathematics. I am talking about modifying the assumption of constant volatility in the Black-Scholes model. I explore different ideas and different parameterizations of the volatility, σ , and then compared them with the original model in order to assess how well they behave. A similar work was examined by Guernsey (2014) who modeled the volatility as a discontinuous function. However, I believe that my project provides a more comprehensive approach, using different parametrization and methods. After presenting a model, the new parametrization of the volatility is then used on the Black-Scholes partial differential equation and is then solved using a Crank-Nicholson scheme. The price returned by the difference scheme is then compared to the market price and the price predicted by the constant volatility model.

The other two parts are studies and practical approaches on other option pricing methods, called the Heston Model and the GARCH option pricing model. Both these models are called stochastic volatility models (SVI).

For the Heston model, I start by describing and analyzing said model. Then, following the steps of Moodley (2005) and Douah (2013), I proceed to discuss, implement and test several calibrations for this model. Finally, I simulate it and compare results with the market price and the price predicted by Black and Scholes.

Finally, for the last part, I introduce an option pricing method using a General Autoregressive Conditional Heteroskedasticity (GARCH) model. I implement the model as proposed by Duan (1993) and Heston & Nandi (2000). Finally, I implement an improvement proposed by Chorro et Al, on which we consider the innovations for the GARCH model to follow a generalized hyperbolic distribution. As with the other two parts, I will compare this model with the market price and with the standard Black-Scholes Model.

I consider that this is a rather interesting approach. Not only it proposes alternatives to the option pricing theory, but it also implements alternatives and their respective improvements. Also, as opposed to the general literature, I shall use a variety of option

chains to test my models. The option chains include companies such as Google, Facebook and Yahoo, as opposed to the widely studied S&P 500. This is important because it is interesting to test these option pricing models in other companies individually, rather than in an economic indicator.