Southwestern Undergraduate Mathematics Research Conference 2013
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ABSTRACTS

Sai Sandilya Babbepalli Venkata (ASU) Mentor: Kathleen Brewer *Applications of Calc 2 to ping pong*
I discuss the math in a simple game of ping pong to show how math is in every small part of our life.

Emalina Bidari and Brandon Samz (NAU) Mentor: Dana Ernst *Structure of braid graphs for reduced words in Coxeter groups of types A and B*
Every element of a Coxeter group can be written as an expression in the generators, and if the number of generators in an expression (including multiplicity) is minimal, we say that the expression is reduced. Every pair of reduced expressions for the same group element are related by a sequence of commutations and so-called braid moves. We say that two reduced expressions are braid equivalent if they are related by a sequence of braid moves. Given a reduced expression r for a group element, we can form the corresponding braid graph whose vertices are the reduced expressions that are braid equivalent to r, and two reduced expressions are connected by an edge if there is a single braid move that converts one reduced expression into the other. In this presentation, we will describe the overall structure of braid graphs for reduced expressions in Coxeter groups of types A and B in terms of products of graphs.

Laney Bowden (CSU) Mentor: Patrick Shipman. *The Least Symmetric Triangle*
"What is the least symmetric triangle?" This question isn't well-posed, but given a quantitative measure of the asymmetry of a triangle, a reasonable answer is: the triangle which maximizes this measure. In turn, if we have a notion of distance on triangle space, the minimum distance to a symmetric triangle (isosceles or degenerate) provides such a measure of asymmetry, and the triangle with the largest such distance can reasonably be called the least symmetric triangle. We used a construction originally developed to study ring polymers like bacterial DNA to define a distance on the space of triangles. Specifically, this will allow us to identify triangles with points on the unit sphere and then use spherical geometry to explicitly find the point on the sphere which is furthest from the isosceles and degenerate triangles: this point then gives us the least symmetric triangle.

Jadyn Breland (NAU) Mentor: Michael Falk *Braid Arrangements and Pointed Multinet Structures, Part I*
We analyze various complex hyperplane arrangements in complex projective space. Our focus will be on line arrangements in the projective plane which carry the structure of a multinet, a multi-arrangement with a partition into three or more equinumerous classes which have equal multiplicities at each inter-class intersection point, and satisfy a connectivity condition. We will explicitly state the necessary conditions for a "pointed multinet structure", on a line arrangement; i.e., a multinet structure obtained from the addition of a line of multiplicity greater than 1. This will be used in conjunction with results of S.Yuzvinsky to examine existence of such structures for the family of braid arrangements and generalizations. The goal is to use a result of A.Dimca to show the pure braid group does not exhibit translated tori in its first characteristic variety.

Samantha Brozak (ASU) Mentor: Wenbo Tang. *Modeling Nutrient-Plankton Reactions in Oceanic Chain Vortices*
The Agulhas system is a key component in global ocean thermohaline circulation and transports warm, briny water from the Indian Ocean to the Southern Atlantic Ocean via the shedding of eddies, known as Agulhas rings. Using direct numerical simulations, we model simple plankton interactions in simulated chain vortices based on Agulhas rings to investigate anomaly residency and dispersion times, stretching, and the dependence of reaction variability on initial conditions.

Alyssa Burgueno (ASU) Mentor: Nancy Childress *p-adic Numbers with an Emphasis on q-Volkenborn Integration and its Applications to Quantum Physics*
Similar to the real numbers, the p-adic fields are completions of the rational numbers. However, distance in this space is determined based on divisibility by p rather than by the traditional absolute value. This gives rise to a peculiar topology which offers significant advantages to discrete functions. Thus, p-adic numbers are well-suited to handle quantum physics phenomena since quantum deals heavily with discrete functions. This project discusses the nature of p-adic numbers and begins to analyze quantum mechanical effects within the p-adic framework.

Tene Carter (UA) Mentor: *Finding Model Parameters from Noisy Data*
This presentation is about finding model parameters from noisy data. In the real world data is usually not simple and easy to read. I present a method of trying to identify patterns in data. The method takes a model and calibrates it in order to try and make predictions. In order to use the method, you send it a vast array of noisy and error-ridden data, and the method identifies a set of parameters that lead to a best fit between model predictions and data.

Ethan Coldren (CSU) Mentor: Henry Adams *A Quadratic-time Algorithm for Persistent Homology in the Number of Points on a Circle*
Let X be a subset of the circle of size n, and let $r \geq 0$. We can construct a simplicial complex on X, by adding $\sigma = \{x_1, x_2, \cdots, x_k\} \subset X$ as a simplex iff for all $1 \leq i, j \leq k$, the distance between x_i and x_j is at most r. Roughly speaking, persistent homology measures how the number of holes varies as r increases. We wish to calculate the l-dimensional persistent homology of this simplicial complex, for all possible values of r. There already exists an algorithm to do this in $O(n^3 \log n)$ time. We present an algorithm to compute it in $O(n^2(l + \log n))$ time.

Gabriella Dalton (UNM) Mentor: Deborah Sulsky *Methods of Estimating Methane Emissions at Albuquerque's Former Los Angeles Landfill*
Amidst the many sources of greenhouse gas emissions in the Unites States, primary landfills are an unsuspecting contributor to high levels of methane emissions exacerbating our global climate crisis. In fact, some estimates have predicted that primary landfills are among the three largest contributors to total methane emissions in the United States. (US EIA) Methane is a common by-product of solid waste decomposition with an estimated Global Warming Potential (GWP) 25 times that of carbon dioxide. The amount of methane released from a given landfill source is highly variable as the stability and consistency of landfill environments are dynamic and dependent on multiple factors; nonetheless, it is important to accurately estimate methane emissions for record-keeping and environmental protection purposes. In the United States, environmental scientists rely heavily on the U.S. Environmental Protection Agency's (EPA) Landfill Gas Emissions Model (LandGEM) Version 3.02 to predict the generation of greenhouse gas levels present in landfills. This study explains the underlying mathematics in LandGEM, and uses empirical data from the Albuquerque Environmental Heath Department (AEHD) for the former Los Angeles landfill located off Paseo Del Norte in Albuquerque, New Mexico to assess the model.

Matthew Daunt (NAU) Mentor: Dr St Laurent Roy. *The Othello Distribution*
Inspired by the board game, Othello is a game in which a 4 by 8 board of disks are each chosen randomly to be either white or black (0 or 1). The color of each disk is an independent Bernoulli random variable with probability of white being .5. The score in the game is determined by the number of lines connected from the top to the bottom of the board. These lines can either be vertical, diagonal, or skew diagonal. I generalized this game to a board of size r by c, allowing multiple colors, and unequal probability of the colors. My research focuses on understanding patterns in the general structure of the probability distribution, including the mean and variance of the score for each version of the game.

Luis Diaz (NAU) Mentor: Michael J. Falk. *Biased Graphs, Matroids, and Multinet Structures*
The goal of this project is to identify which bias matroids support multinet structures. In this talk we focus on bias matroids that arise from voltage graphs. We introduce the relevant notions, and examine conditions, for voltage graphs to support neighborly partitions, a necessary condition for the existence of multinet structures.

Alyssa Lyn Fortier (UA) Mentor: Bruce Bayly *Its a Small World After All: Epidemics and Immunization on Euclidean-Distance-Preferred Small-World Networks*
A networks topology determines how data, such as a chain email, computer virus, or even gossip, spreads across its elements. Accurately modeling this spread becomes especially critical when considering the progression of a deadly disease. Few previous computational analyses have incorporated Euclidean distance into epidemic spread models, and even fewer have integrated biologically important phenomena such as pathogen mutation. Using a Euclidean-distance-preferred model, we investigated epidemic dynamics and immunization strategies with and without pathogen mutation. We determined that the optimal immunization time and radius depend on the clustering coefficient of the underlying network. In addition, we concluded that the original strain flourishes on less-clustered networks, while the mutated strain spreads more readily on highly-clustered networks. These results demonstrate the sensitivity of epidemic spread to network topology and establish the need for highly specific models to achieve predictive power.

Kody Gelvin (NAU) Mentor: Derek Sonderegger *Comparing function estimation techniques using data from an oxidative stress experiment Part 1* Blood protein levels can change dramatically over time in response to stimulus. In this study, two function estimation techniques (piecewise interpolation and generalize additive models) are compared via simulation. Several different response functions are considered to model what the possible relationship between blood protein levels and time is. Within the function, noise values were added with epsilon values of .1, .2, and .3 to model different error levels. Lastly, it is necessary to compare the results of the piecewise interpolation process and generalized additive models simultaneously, in respect to the true value.

Marcela Gutierrez (NAU) Mentor: Jeff Rushall *An Infinite Tree of Primitive Pythagorean Quadruples, Part I*
A primitive Pythagorean triple is a 3-tuple of natural numbers sharing no nontrivial common factors that satisfies the Pythagorean Theorem. Hall (1970) and Price (2008) found distinct perfect infinite ternary trees whose vertex sets are precisely all primitive Pythagorean triples. Using elementary tools, we will construct an infinite tree whose vertex set consists of all nonnegative primitive Pythagorean quadruples—i.e., 4-tuples (d, a, b, c) of natural numbers having no nontrivial common factors that satisfy $d^2 = a^2 + b^2 + c^2$. We will also present some interesting subtrees with curious properties. This is part one of two talks.

Robert Hollenback (ASU). Mentor: Naala Brewer. *"Pi" Equivalence for a Triangle*
I came up with the a constant for ""Pi"" of an equilateral triangle (called ""Triangle Pi"") out of necessity. I built 4, 8, and 20 sided dice using equilateral triangle for the face pattern. I drew entire pages of triangles to make my dice with knowing the height of the triangles I am trying to draw is a necessity for 2 reasons: 1) It makes it quicker and easier to do the layout and 2) I need to be able to draw the triangles without a protractor. I noticed that with all sizes of triangular faces, there was always a constant value for the ratio of the perimeter and the height of an equilateral triangle. After taking PreCalculus with the ASU classes at our facility, I was able to prove my conjecture. I will present this proof in a power point presentation and I will expand this proof to any n-sided equilateral polygon. As n goes to infinity, the proof gives rise to the value of ""Pi"" for a circle."

Andrew Hollis (UNM) Mentor: James Degnan. *Clustering and Summarization for Large Document Sets*
The intelligence community is awash in large quantities of data, and much of this data is text data. The need for tools that can help analysts quickly separate useful texts from less useful texts is becoming ever more critical. The goal of this paper is to explore a variety of clustering and summarization algorithms that can be used together to create systems that allow analysts to quickly get an idea of the major groups in a document collection and the issues discussed within these document groups. In order to evaluate the effectiveness of these algorithms we compare their performance to clustering and summarization results generated by human analysts.

David Hurowitz (UA) Mentor: Rault. *The ABCs You Were Never Taught In School: The ABC Conjecture*
The ABC Conjecture was first proposed by J. Oesterl and D.W. Masser in 1985. It relates the integer solutions of C = A + B with GCD(A,B,C) = 1 to the prime factors of the product of ABC. A successful proof of the ABC Conjecture would lead to new proofs of several important theorems and conjectures in mathematics, including Fermats Last Theorem. The presenter will give a full formulation of the ABC Conjecture and describe the ramifications of a successful proof.

Joshua de Jong (CSU) Mentor: Chris Peterson *Labeled Adjacency Matrices*
A variation on adjacency matrices called labeled adjacency matrices M encodes information about the edges of graph G as variables. By calculating M^n where n is the length of a path, M^n encodes all paths of length n as polynomials. These polynomials can be used to answer questions such as the following: How many paths of length n use the same edge twice? How many paths of length n use a defined set of edges? How connected are two vertices on G?

Ryan Kelly (NAU). Mentor: Jim Swift. *Simulation and Investigation of N-body Systems*
Abstract: A complete theory of quantum gravity would revolutionize our understanding of the universe we live in. By simulating various universal forces we can visually investigate the nature of these forces. Using Unity 3D, a gaming and physics programming platform, and the Runge-Kutta 4 approximation method, I modeled Newtonian Gravitation and found stable particle orbits. Then, as a first order correction, I added a propagation speed to gravitation to investigate its potential effects on orbital stability and precession. I also modeled several other forces between objects: Hookes law to investigate molecular vibrations and normal modes, as well as Coulomb's law and magnetism to investigate the motion of classical charged particles.

Through these simulations, we are able to see that objects which interact through retarded gravitational potential cannot produce stable orbits. We can also visualize normal modes of molecular models and begin to observe large scale effects of classical electricity and magnetism. Although I did not directly investigate any modern quantum phenomena, using numerical approximations to predict the motion of particles is a promising method for investigating various implications of our current theories. Possible further research would be to simulate quantum electromagnetism to study quantum gravitation.

Kaitlyn Lee (NAU) Mentor: Jeff Rushall *New Complex Hadamard Matrix Constructions Part I*
A matrix H that satisfies $HH^T = nI_n$ and has only ± 1 entries is said to be Hadamard. Hadamard matrices have many applications and play important roles in signal processing, coding theory and cryptography. Both Scarpis (1896) and Paley (1933) constructed new infinite classes of Hadamard matrices. In these two talks, we will modify the approaches of Scarpis and Paley to construct quaternary complex Hadamard matrices - that is, matrices that satisfy $H \cdot H^T = nI_n$ and have entries from the set $\{\pm 1, \pm i\}$.

Viola McCarty (NAU) Mentor: Jeff Rushall *An Infinite Tree of Primitive Pythagorean Quadruples, Part II*
A primitive Pythagorean triple is a 3-tuple of natural numbers sharing no nontrivial common factors that satisfies the Pythagorean Theorem. Hall (1970) and Price (2008) found distinct perfect infinite ternary trees whose vertex sets are precisely all primitive Pythagorean triples. Using elementary tools, we will construct an infinite tree whose vertex set consists of all nonnegative primitive Pythagorean quadruples—i.e., 4-tuples (d, a, b, c) of natural numbers having no nontrivial common factors that satisfy $d^2 = a^2 + b^2 + c^2$. We will also present some interesting subtrees with curious properties. This part two of two talks.

William van Noordt (CSU) Mentor: Patrick Shipman *Mesh-Morphing on a Rectangular Domain via an Iterative Gradient-Ascent Algorithm*
When numerically estimating solutions to boundary value problems, three components are critical to solution quality: the numerical methods used to estimate aspects of the governing equations, proper specification of boundary conditions, and the discretization, or mesh, of the domain upon which the solution is to be approximated. This method seeks to provide a way to automatically improve the quality of a mesh based on an initial estimate of a solution and an iterative gradient-ascent algorithm. The method was shown to be effective in improving solution accuracy for minimal surfaces and other problems, but was not an effective means of solution improvement in some cases. Further discussion is provided on the method, future improvements, as well as applications.

Jaxon Quillen (NAU) Mentor: Derek Sonderegger *Comparing function estimation techniques using data from an oxidative stress experiment Part 2*
Blood protein levels can change dramatically over time in response to stimulus. In this study, two function estimation techniques (piecewise interpolation and generalize additive models) are compared via simulation. Several different response functions are considered to model what the possible relationship between blood protein levels and time is. Within the function, noise values were added with epsilon values of .1, .2, and .3 to model different error levels. Lastly, it is necessary to compare the results of the piecewise interpolation process and generalized additive models simultaneously, in respect to the true value.

Ari Rappaport (UNM) Mentor: Jehanzeb Chaudhry. *Exploration of Black Box Multigrid for Linear Systems Modeling Resistive MHD*
Discretizing and numerically solving partial differential equations (PDEs) is of great interests to many applied mathematicians, engineers and scientists. In this project, we are primarily interested in a resistive magnetohydrodynamics (MHD) PDE model which can be used to simulate plasma physics. We explore the robustness of a multilevel (Black Box Multigrid) method for the efficient numerical solution of this system.

Michael Rozinski (NAU) Mentor: Terence Blows *Constructing a Multiplayer Rating System Part 1*
The Elo rating system is most widely known for its use in tournament chess. This system produces numerical ratings which represent the measure of a player's skill set. The system, named after its creator Arpad Elo, is utilized for many purposes, including the pairing of players in a tournament and predicting the outcome of a two-player game. We will outline all relevant characteristics of Elo's rating system and examine key features of the algorithm it uses. We will then deduce potential issues by cross-referencing current multi-player rating systems that use rudimentary paired comparisons, and construct a fairness criteria that addresses these issues. With this fairness criteria, we will assemble a rating system that extends Elo's two-player algorithm to multi-players and discuss its success in accurately predicting a players ability.

Mason Sargent (NAU) Mentor: Jeff Rushall *New Complex Hadamard Matrix Constructions, Part 2* (see abstract Lee)

Bibiana Seng (UNM) Mentor: James Degnan *Application of Statistical Clustering to Characterize Ocean Sites for Wave Energy Converter Placement*
The reliability of a Wave Ocean Converter (WEC) design is heavily reliant on the extreme ocean conditions at a chosen deployment site. Current analysis of ocean sites uses statistical contours to determine expected values for the significant wave height and energy period for a given time interval, known as a return period. The statistical contour method is currently run on a site-by-site basis; as such it can be difficult to detect overall patterns and expected values in large regions. My presentation will cover the application of a statistical clustering method that uses the Mean Shift algorithm to group together similar sites, and identify trends in the clusters. I will also discuss how these results were inferred. From there, I will discuss how this method can assist in choosing the deployment site for a WEC design, and how this could be further refined and researched in the future.

Justin Sima (NAU) Mentor: Jeff Rushall *Prime Vertex Labelings for Perfect Ternary Trees, I*
A prime vertex labeling on a simple connected graph G(V,E) with k vertices is an injective function f from V to 1,2,,k such that adjacent vertices are assigned relatively prime labels. Our project involves finding a prime vertex labeling for all perfect ternary trees, which has been an open problem since 1994. Note that this is the first of two talks on prime vertex labelings of perfect ternary trees.

Kaitlynn Trimble (ASU) Mentor: Naala Brewer. *An Application of Geometric Series: Annuities*
One example of applying geometric series in the real-world can be seen through annuities. An annuity is a geometric series where there is a string of payments that are made usually in equal intervals of time and amounts. Remember that a geometric series is a sequence of numbers where the initial term, a is multiplied by a common ratio, r, in order to get the subsequent numbers in the sequence. For an annuity, the initial term and the common ratio are the same value that is represented by 'v'. Therefore, by comparing the two one can gain a better understanding of what an annuity is and how they function. In other words, a geometric series is a simple way to define an otherwise complicated subject.

Javier Urcuyo (ASU) Mentor: Dr. Abba Gumel. *Data-driven model for mosquito population dynamics and the spread of Zika virus using environmental inputs*
Many worldwide vector-borne diseases, including Zika virus, are transmitted through mosquitoes. Understanding the dynamics of Aedes mosquitoes is crucial to controlling and mitigating Zika virus in human populations. Life-cycle, development, biting rate, and survival of mosquitoes are all affected by changes in climatic variables such as temperature and precipitation. Laboratory experiments are currently being conducted to study these effects. By using the biological data collected, as well as historical case data and environmental data, an ordinary differential equation model was created to estimate the abundance of each mosquito species in the population.

Riley Waechter (NAU) Mentor: Jeff Rushall *Prime Vertex Labelings for Perfect Ternary Trees, II*
A prime vertex labeling on a simple connected graph G(V,E) with k vertices is an injective function f from V to 1,2,,k such that adjacent vertices are assigned relatively prime labels. Our project involves finding a prime vertex labeling for all perfect ternary trees, which has been an open problem since 1994.

Sean Willmot (CSU) Mentor: Henry Adams *On Vietoris-Rips Complexes of Planar Curves*
Given a finite sample from a metric space, a Vietoris–Rips complex is a way to thicken that sample into a larger space. While the shapes of these complexes are very hard to describe for general metric spaces, these shapes are known for subsets of the circle. In this talk, we show that if C is any closed convex curve in the plane that contains its evolute, then the shape of the Vietoris–Rips complex of points from C can similarly be characterized.

Ryan Wood (NAU) Mentor: *On Matrices Generated by Higher Order Reciprocity Symbols*
The use of quadratic residues to construct matrices with specific determinant values is a familiar problem with connections to many areas of mathematics and statistics. Using the results of Paley and Vsemirnov as a guide, we will present results on the use of higher order reciprocity symbols to construct matrices with interesting and predictable determinants.

Jordan Wright (NAU) Mentor: Michael Falk *Braid Arrangements and Pointed Multinet Structures, Part II* (See abstract Breland)

Jordan Wright (NAU) Mentor: Terence Blows *Constructing a Multiplayer Rating System, Part II* (See Abstract Rozinski)