Nonlinearity management in a dispersion-managed system

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We propose using a nonlinear phase-shift interferometric converter (NPSIC), a new device, for lumped compensation for nonlinearity in optical fibers. The NPSIC is a nonlinear analog of the Mach–Zehnder interferometer and provides a way to control the sign of the nonlinear phase shift. We investigate a potential use of the NPSIC for compensation for nonlinearity to develop a dispersion-managed system that is closer to an ideal linear system. More importantly, the NPSIC can be used to essentially improve single-channel capacity in the nonlinear regime.

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The recent invention[1] and testing[2–5] of the dispersion-management technique demonstrated the effectiveness of this approach for high-speed communications. Optical pulse dynamics in fiber links with dispersion management is governed by the nonlinear Schrödinger equation with periodic coefficients:

$$iu_z + d(z)u_{tt} + \sigma(z)|u|^2u = 0\,, \tag{1}$$

where z is the propagation distance, u is an optical pulse amplitude, $d(z) \equiv -{}^1\!/_2\beta_2(z)$, $\beta_2(z)$ is a first-order group-velocity dispersion, $\sigma(z) = (2\pi n_2)/[\lambda_0 A_{\text{eff}}(z)]$ is the nonlinear coefficient, n_2 is the nonlinear refractive index, $\lambda_0 = 1.55\ \mu\text{m}$ is the carrier wavelength, and the effective area of the fiber, A_{eff}, in general, depends on z.

On short scales, the dispersion-managed (DM) system is practically linear. Linear transmission in an optical fiber is limited by a nonlinear distance, $z < z_{\text{nl}} \equiv (\sigma|u_0|^2)^{-1}$, that is determined by Kerr nonlinearity σ and characteristic pulse power $|u_0|^2$. The characteristic power cannot be chosen to be too small if an appropriate value of the signal-to-noise ratio is to be maintained. It is natural to attempt to extend the scale of the applicability of the linear regime. This extension can be achieved through the use of a new optical fiber with lower Kerr nonlinearity.[6,7] Another obvious approach is nonlinearity management, which was considered in Ref. 8. However, the semiconductor material waveguides with negative Kerr nonlinearity that were proposed as an element for compensation for nonlinear phase shift are not currently practical. In this Letter we consider lumped compensation of the nonlinearity, the analog of lumped compensation for the chromatic dispersion by means of chirped fiber gratings. We suggest the use of a nonlinear phase shift interferometric converter (NPSIC), as presented in Fig. 1. The NPSIC consists of silica-based Fiber 1, highly nonlinear Fiber 2 (which can be chalcogenide glass based, with a Kerr coefficient ~400 times, or even much more than, that of silica; see, e.g., Ref. 9), a linear amplifier, A, with amplitude amplification coefficient G, and direction Couplers 1 and 2. It is assumed that an optical terminator is installed at the end of Fiber 2 after Coupler 2 to prevent beam reflection from the end of Fiber 2. The incident light, with amplitude Gu_0, is split by direction Coupler 1 into two beams with amplitudes a_1Gu_0 and $a_2Gu_0\exp(i\pi/2)(a_1 > 0, a_2 > 0)$, corresponding to Fibers 1 and 2, respectively. We assume that total power is conserved, ${a_1}^2 + {a_2}^2 = 1$, but a subsequent consideration can be easily generalized so that the directional couplers' and the fibers' losses are included. The extra phase $\pi/2$ in the second fiber is due to light splitting in the directional coupler (see, e.g., Ref. 10). The optical lengths l_1 and l_2 in Fibers 1 and 2 between Couplers 1 and 2 are chosen so that they provide a zero phase difference at the input of Coupler 2, $n_{\text{eff},1}l_1 - n_{\text{eff},2}l_2 = 0$, where $n_{\text{eff},1}$ and $n_{\text{eff},2}$ are effective linear refraction indices in Fibers 1 and 2, respectively. We assume that l_1 and l_2 are small enough that we can neglect the influence of dispersion in both fibers and the nonlinear phase shift in Fiber 1. Thus the amplitudes of optical beams at the input of Coupler 2 are given by a_1Gu_0 and $a_2Gu_0\exp(i\pi/2 + i\phi_{\text{nl}})$, where nonlinear phase shift $\phi_{\text{nl}} = \sigma_{\text{hnl}}{a_2}^2G^2|u_0|^2l_2$ and σ_{hnl} correspond to the value of σ in highly nonlinear Fiber 2. Assuming that the propagation constants are the same for symmetric and antisymmetric modes of the coupler, we can write the coupled-wave equation that describes mode evolution in directional couplers as[10]

$$(u_1)_z = i\kappa u_2, \qquad (u_2)_z = i\kappa u_1\,, \tag{2}$$

where

$$u_1(z_0) = a_1Gu_0, \qquad u_2(z_0) = a_2Gu_0\exp(i\pi/2 + i\phi_{\text{nl}}),$$
$$u_{1\text{out}} = u_1(z_{\text{out}})\,, \tag{3}$$

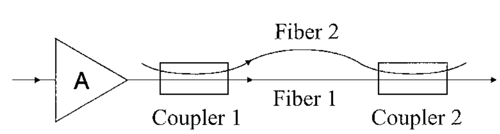

Fig. 1. Schematic of the NPSIC.

z_0 and z_{out} are the coordinates of Coupler 2 input and output, respectively; u_{1out} is the optical beam amplitude in Fiber 1 at the exit of the Coupler 2 and κ is a coupling coefficient that is assumed to be real in the lossless model. The solution of Eqs. (2) and (3) shows that the NPSIC converts u_0 into output signal u_{1out}:

$$u_{1out} = Gu_0[a_1 \cos \phi_c - a_2 \exp(i\phi_{nl})\sin \phi_c], \quad (4)$$

where $\phi_c = \kappa(z_{out} - z_0)$. We assume that $\phi_{nl} \equiv \sigma_{hnl} a_2{}^2 G^2 |u_0|^2 l_2 \ll 1$ and expand $\exp(i\phi_{nl})$ in Eq. (4). Neglecting the $O(\phi_{nl}{}^2)$ terms results in

$$u_{1out} = u_{lin}\left(1 - \frac{i\sigma_{hnl} a_2{}^3 G^2 |u_0|^2 l_2 \sin \phi_c}{a_1 \cos \phi_c - a_2 \sin \phi_c}\right), \quad (5)$$

where $u_{lin} \equiv Gu_0(a_1 \cos \phi_c - a_2 \sin \phi_c)$ is the output amplitude, u_{1out}, of the linear system. Thus, by changing NPSIC parameters a_1, ϕ_c, l, and G, one can control the sign and magnitude of the nonlinear phase shift. NPSIC can be considered as a nonlinear version of the Mach–Zehnder interferometer. As a typical example, we take $G = \sqrt{50}$, $a_1 = 1/3$, and $\phi_c = 0.2$, which results in $u_{1out} \simeq u_0(1.0 - i60\sigma_{hnl}|u_0|^2 l_2)$. Thus, to compensate completely for a nonlinear phase shift $\sigma|u_0|^2 L_1$ in the $L_1 = 40$ km line of a silica fiber, we need to use a NPSIC with $l_2 = 1.6$ m of a highly nonlinear fiber, provided that $\sigma_{hnl} = 400$ (for this value of σ_{hnl} the nonlinear absorption is negligible in currently available chalcogenide glasses[9]).

This estimate is true if the power, $|u|^2$, is constant throughout the propagation. In practice, the power is not constant because of the fiber's chromatic dispersion and losses. Therefore, nonlinearity compensators must be distributed along the fiber with a separation distance less than the dispersion length, $z_{disp} \equiv \tau^2/d$, and loss length z_{loss}. Here τ is a typical pulse width. It is shown below that the effective length of the nonlinearity is increased as $z_{eff,nl} \sim N^2 z_{nl}$, where N is the number of lumped NPSICs on the dispersion-map period. However, we demonstrate that for short pulses corresponding to strong dispersion management this approach is efficient only at a relatively large value of N. Nevertheless, this method displays good performance in the nonlinear regime, as, for example, inserting only two compensating elements on the period of the dispersion map could increase the bit rate per channel by a factor of 2.

Consider a DM system with stepwise periodic dispersion variation: $d(z) = d_0 + \tilde{d}(z)$, $\tilde{d}(z) = d_1$ for $0 < z + mL < L_1$ (standard monomode fiber) and $\tilde{d}(z) = d_2$ for $L_1 < z + mL < L_1 + L_2$ (dispersion-compensating fiber). Here d_0 is the path-averaged dispersion, d_1 and d_2 are the amplitudes of dispersion variation subject to a condition $d_1 L_1 + d_2 L_2 = 0$, $L \equiv L_1 + L_2$ is a dispersion-compensation period, and m is an arbitrary integer number. $\sigma(z) = \sigma_1$ for $0 < z + mL < L_1$ and $\sigma(z) = \sigma_2$ for $L_1 < z + mL < L_1 + L_2$, corresponding to standard monomode and dispersion-compensating fiber. We suppose that the NPSIC units are located inside and at the ends of a dispersion-compensated fiber at points $z_n = mL + L_1 + nL_2/N$, $n = 0, \ldots, N$, $z_0 = mL + L_1 < z_1 < \ldots < z_N = (m + 1)L$. At these points the value of $u(t, z)$ experiences a jump according to Eq. (5):

$$u(t, z)|_{z=z_{m,n}+0} = (u - iz_{eff,n}|u|^2 u)|_{z=z_{m,n}-0}, \quad (6)$$

where $z_{m,n} \equiv mL + L_1 + nL_2/N$; $z_{m,n} - 0$ and $z_{m,n} + 0$ are the coordinate values just before and after the jump, respectively; parameters a_1, ϕ_c, l, and G are chosen to provide $u_{lin} = u(t, z)|_{z=z_{m,n}-0}$ and $\sigma_{hnl} a_2{}^3 G^3 l_2 \sin \phi_c \equiv z_{eff,n} \equiv (\sigma_1 L_1 + \sigma_2 L_2)/N$ for $n = 1, \ldots, N - 1$ and $z_{eff,0} \equiv z_{eff,N} \equiv (\sigma_1 L_1 + \sigma_2 L_2)/(2N)$. Here the term $-(\sigma_1 L_1 + \sigma_2 L_2)$ provides the compensation for the nonlinear phase shift in both standard monomode and dispersion-compensating fibers.

Assuming that the nonlinearity is small, $z_{nl} \gg z_{disp}$, $z_{nl} \gg L$, where L is a dispersion-map period, one can express u in the Fourier domain as a product of an exact solution of the linear part of Eq. (1), corresponding to mean-free dispersion $\tilde{d}(z)$, on a slow function $\hat{\psi}(\omega, z)$: $\hat{u}(\omega, z) \equiv \hat{\psi}(\omega, z)\exp[-i\omega^2 \int_{z_0}^{z} \tilde{d}(z')\mathrm{d}z']$,[11] where $\hat{u}(\omega, z) = \int_{-\infty}^{\infty} u(t, z)\exp(i\omega t)\mathrm{d}t$. $\hat{\psi}$ is a slow function of z on a scale L, which allows us to integrate Eq. (1) over the period L, neglecting the slow dependence of $\hat{\psi}$ on z, and by taking into account the jumps in Eq. (6) we get

$$i\hat{\psi}[\omega, (m+1)L] - i\hat{\psi}(\omega, mL) - L\omega^2 d_0 \hat{\psi}(\omega, mL) + \frac{1}{(2\pi)^2}\int \hat{\psi}(\omega_1, mL)\hat{\psi}(\omega_2, mL)\hat{\psi}^*(\omega_3, mL) \times K_{tot}(\Delta)\delta(\omega_1 + \omega_2 - \omega - \omega_3)\mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{d}\omega_3 = 0, \quad (7)$$

where $s \equiv d_1 L_1$, $\Delta \equiv \omega_1{}^2 + \omega_2{}^2 - \omega^2 - \omega_3{}^2$, and $K_{tot}(\Delta) \equiv K_1(\Delta) + K_2(\Delta)$. A kernel $K_1(\Delta)$ is equal to the usual kernel of the path-averaged equation[11]

$$K_1(\Delta) = \sigma_L \sin x/x, \qquad x \equiv s\Delta/2, \qquad \sigma_L \equiv \sigma_1 L_1 + \sigma_2 L_2, \quad (8)$$

and $K_2(\Delta)$ accounts for the jumps in Eq. (6):

$$K_2(\Delta) = -\sigma_L[\sin x \cot(x/N)]/N. \quad (9)$$

Suppose that the FWHM τ of $\psi(0, t)$ is subject to the condition $s/(2N\tau^2) \ll 1$. Then, $K_{tot}(\Delta)$ can be rewritten as

$$K_{tot}(\Delta) = \sigma_L(\sin x/x)[x^2/(3N^2) + O(x^4/N^4)]. \quad (10)$$

Thus, comparing Eq. (10) with kernel (8) of the usual path-averaged equation, we get an extra small factor $(s\Delta/2N)^2/3 \sim L^2/(Nz_{disp})^2$, provided that N is big enough to ensure the condition $L/Nz_{disp} \ll 1$. For short pulses N should be a large number for the system to approach the linear regime (where the nonlinear integral term is absent). Figure 2 shows the dependence of the FWHM τ_{out} obtained after propagation of the initial zero-chirp Gaussian pulse with $\tau_{ini} = 10$ ps over a typical transoceanic distance

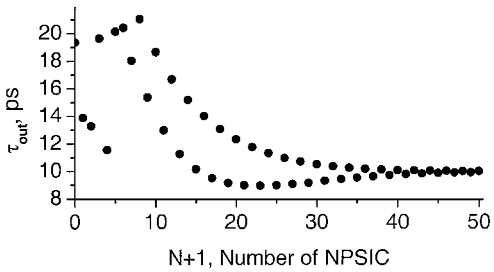

Fig. 2. FWHM τ_{out} after pulse propagation over 10^4 km versus a number $N + 1$ of NPSIC units.

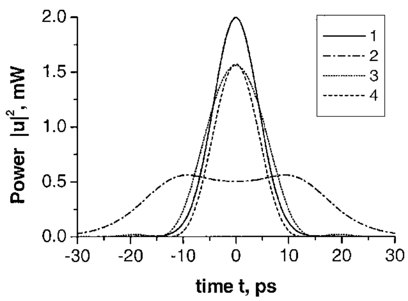

Fig. 3. Power distributions: 1, initial Gaussian pulse; 2, result of pulse propagation over 10^4 km in a DM system with no NPSIC; 3, result of pulse propagation in a DM system with two NPSIC units for each period L; 4, MDM soliton.

10^4 km on the number of NPSIC units, $N + 1$. This dependence is obtained by numerical integration of nonlinear Schrödinger equation (1). The pulse is launched into the DM fiber at $z = L_1/2$ with peak power $|u|^2 = 2$ mW. The DM system parameters are $d_0 = 0$, $d_1 = 10.0$ ps^2/km, $d_2 = -50.8$ ps^2/km, $L_1 = 40$ km, $L_2 = -d_1L_1/d_2$, $\sigma_1 = 0.0013$ (km mW)$^{-1}$, $\sigma_2 = 0.00405$ (km mW)$^{-1}$. It can be seen that the system can be considered approximately linear for $N \gtrsim 30$, in which case the nonlinear correction to τ_{out} is less than 5%.

A strong variation of τ_{out} in Fig. 2 for small N is due to the attraction of the initial Gaussian pulse to soliton solution $\hat{\psi}(\omega, mL) \equiv \hat{\psi}_0(\omega)\exp(imL\lambda)$ of path-averaged equation (7). Here λ is a soliton propagation constant. We refer to this soliton solution as the modified DM soliton (MDM soliton) by analogy with the DM soliton, which corresponds to the usual path-average equation (without nonlinearity management). Without nonlinearity compensation, $K_2(\Delta) = 0$, we recover the usual DM soliton of the path-averaged equation,[11] where the DM soliton width τ_{DM} depends on s only for $d_0 \to 0$ (see, e.g., Ref. 12). For the above-mentioned system parameters, $\tau_{DM} \simeq 21$ ps. Any shorter pulses in this case experience strong distortion because of the nonlinearity. Curve 2 in Fig. 3 shows the pulse-power distribution after 10-km propagation of the initial Gaussian pulse, with $\tau_{ini} = 10$ ps (curve 1). Using NPSIC units, we can control $K_2(\Delta)$ and thus change τ_{MDM} of the MDM soliton. Curve 3 shows the pulse-power distribution after 10^4-km propagation. The transmission system consists of two NPSICs for each DM period L, located at the ends of the dispersion-compensating fiber, with $z_{eff,0} = z_{eff,1} = f(\sigma_1L_1 + \sigma_2L_2)/2$, where $f = 3/2$. One can see that curve 3 is close to both the initial Gaussian pulse (curve 1) and the MDM soliton (curve 4). The MDM soliton was obtained by numerical iteration of Eq. (7), with $\hat{\psi}(\omega, mL) = \hat{\psi}_0(\omega)\exp(imL\lambda)$, $\lambda = 0.00028$ km^{-1}. The numerical iteration scheme is similar to the one used in Ref. 13 for the usual path-averaged equation. By changing factor f, one can control τ_{MDM} of the MDM soliton. Note that, for $f > 1$, which corresponds to a negative average nonlinearity of the total system, the numerical iteration scheme finally diverges, indicating that the MDM soliton is a rather long-lived quasi-stable structure, which, however, serves as an attractor of the pulse dynamics on a scale of $\sim 10^4$ km.

The use of NPSIC elements would allow one to construct a nearly linear fiber transmission system. However, this system requires many NPSIC elements. On the other hand, the use of NPSIC elements offers the better advantages to fiber links operating in the nonlinear regime. In particular, only a few elements per dispersion-map period could dramatically reduce the pulse width and potentially increase the bit rate.

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