Nonlinear Waves and Singularities in Optics, Hydrodynamics and Plasmas

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PREFACE

Waves dynamics is one of the most interesting and appealing problems in applied mathematics and physics. We encounter waves in all areas of our everyday lives, from waves on the surface of a lake or a pool and sound waves to the electromagnetic waves propagation in ionosphere and plasma excitations on the sun. In wast majority of interesting cases the problem of wave propagation can be solved not only in the linear approximation but also with nonlinear effects taken into account, due to powerful tools of modern applied mathematics and theoretical physics. These approaches together with rapidly emerging computational power leads to new amazing advances in the study of waves dynamics in different media. Common approaches stimulate intensive interchange of ideas in the field which accelerates the development of the wave dynamics even further. Our minisymposium is devoted to new advances in the theory of waves and demonstrates vividly the similarity of approaches in a broad spectrum of important applications.

Nonlinear waves on the surface of a fluid are one of the most well known and complex phenomena in nature. Mature ocean waves and ripples on the surface of the tea in a pot, for example, can be described by very similar equations. Both these phenomena are substantially nonlinear, but the wave amplitude is usually significantly less than the wavelength. Under this condition, waves are weakly nonlinear. In the middle of 20th century statistical theory of water waves, based on the the kinetic equation for waves derived by Hasselmann and solutions of this equation obtained from Zakharov's theory of wave (or weak) turbulence. These solutions are stationary Kolmogorov solutions of the kinetic equation corresponding to flux of energy from large to small scales (direct cascade) and flux of wave action (waves "number") from small to large scales (inverse cascade). Now the kinetic equation is a base tool for wave forecasting.

May be the most promising way to check conjectures of the waves turbulence theory is a numerical experiment. In the case of direct numerical simulation we have the highest possible control on the parameters of experiments and all information about the wave field. At the same time all this data is given at the cost of the enormous computational complexity. Fast growth of computational power and development of computational algorithms allowed direct numerical simulation of the surface gravity waves, starting from the simulations of the swell evolution to the isotropic turbulence simulation. There is a hope, that this approach together with confirmation of conjectures of the weak turbulent theory will allow us to explain phenomena observed in experimental wave tanks as well as in the ocean.

Nonlinear Schrödinger equation (NLS) is among best illustrations of successful application of applied mathematics as a tool to analyze various nonlinear phenomena ranging from optical communications and Bose condensation to ocean waves. Solutions of nonlinear equations usually result in the formation of singularities, coherent structures or solitary waves. Examples of the corresponding phenomena can be observed in filamentation of laser beams in nonlinear media, wave breaking in hydrodynamics, collapse, and Langmuir waves in plasmas.

NLS in dimension one (1D) is a basis for the description of modern optical fiber communication systems. A key technological tool for development of ultrafast high-bit-rate optical communication lines is a dispersion management. A dispersion-managed optical fiber is described by NLS with periodic variation of dispersion along an optical line which dramatically reduces pulse broadening. Solitary wave solutions in such systems are important information carriers in optical lines.

NLS in 2D describes a stationary self-focusing of light in optical media with Kerr nonlinearity, Bose-Einstein condensate in plane geometry as well as a self-focusing in plasma. Due to self-focusing the amplitude of an NLS solution tends to infinity after finite distance of propagation, representing a blow up phenomenon. The blow-up is

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accompanied by a dramatic contraction of the spatial size of solution, which is called collapse. Near the collapse point NLS loses its applicability and there is usually a qualitative change in the underlying nonlinear phenomena, the new mechanisms become important such as dissipation, inelastic two- and three-body collisions, which can cause a loss of atoms from the Bose-Einstein condensate, breakdown of slow envelope approximation in nonlinear optical media and plasma density depletion. If optical power is very large in optics or if a number of particles in the Bose-Einstein condensate significantly exceeds the critical number, then collapse turbulence occurs with random positions of collapses in space and time. A study of collapse regularization is important in describing collapse turbulence and to be able to go beyond an individual collapse event.

3D NLS is a model for non-stationary self-focusing of light in optical media with Kerr nonlinearity, Bose-Einstein condensate in 3D geometry and laser-plasma interactions. Self-focusing results in collapse, which is of a different type as compare with 2D, and it is called weak collapse. NLS in 3D is a model for ultrashort pulse (optical bullet) that is a key for current developments in microfabrication of photonics devices by means of intense femtosecond laser pulses. Regularization of collapse for such pulses requires an extension of the model beyond NLS and a study of the full set of Maxwell's equations. Maxwell's equations are also necessary to study ultrashort light propagation through thin films and guided-wave metamaterials.

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