

Fall 2026. MATH 579 - 002: Nonlinear Partial Differential Equations with Applications to Fluid Dynamics, Quantum Fluids, Optics, Plasmas and Biology

Instructor: Pavel Lushnikov

TR 12:30-13:45 pm. Room : TBA

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Course web page:

<https://math.unm.edu/~plushnik/teaching/math579nonlinearwaves2026fall/math579syllabus2026fall.pdf>

We surrounded by waves in our everyday life such as light, sound and oceanic waves. Linear waves are described by linear partial differential equations (PDE). Waves become nonlinear when we can not consider their amplitudes being arbitrary small. Contrary to linear waves, nonlinear waves are described by nonlinear PDE, they interact with each other and often form new structures. This course will be focused on modern theory of nonlinear PDE with application to nonlinear optics and lasers, Bose-Einstein condensation, fluid dynamics, water waves, optical fiber communications, electrical engineering and biology. The introduction to the Hamiltonian formulation of nonlinear waves will be given. We will derive universal models of nonlinear waves including Burgers equation, Korteweg-de Vries equation, nonlinear Schrödinger equation and Kadomtsev-Petviashvili equation. Brief introduction to the spectral method for numerical solution of partial differential equations will be also given. The course is intended for mathematicians, physicists, computer scientists and engineers. Topics covered will include:

1. Linear waves in optics and hydrodynamics. Phase and group velocity.
2. Nonlinear interaction of waves. Nonlinear Schrödinger equation (NLSE). Derivations of NLSE in nonlinear optics, fluid dynamics, quantum fluids (Bose-Einstein condensates) and plasmas.
3. Introduction to the inverse scattering transform. Integrability of nonlinear Schrödinger equation.
4. Nonlinear light waves. Nonlinear optics. Kerr nonlinearity.
5. Optical fiber communications and solitons.
6. Nonlinear waves in more than one dimension. Kadomtsev-Petviashvili equation. Nonlinear Schrödinger equation in dimension two. Catastrophic self-focusing of laser beams and breaking of oceanic waves. Wave collapse and formation of singularities.
7. Vibrating nonlinear string. Fermi-Pasta-Ulam problem. Multiscale expansion.
8. Basic equations of hydrodynamics. Euler equations. Navier-Stokes equations.
9. Dynamics of bacterial colonies, Keller-Segel equation.
10. Numerical solution of partial differential equations. Fast Fourier Transform. Spectral and pseudospectral methods.

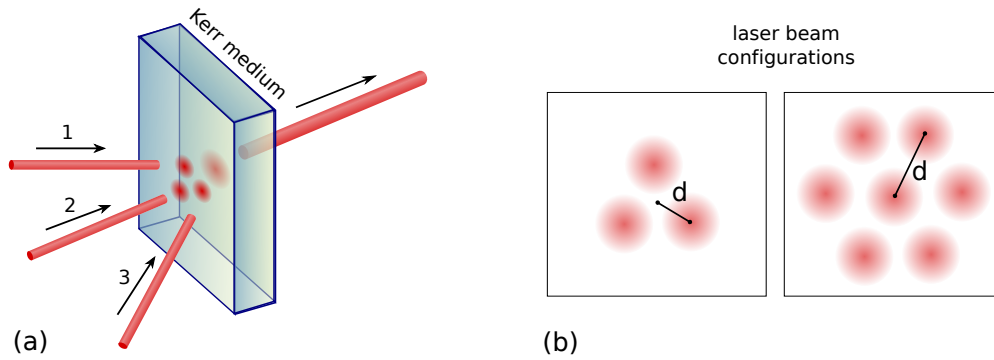


Figure 1: A schematic of the nonlinear beam combining through collapse. (a) An array of beams with non-correlated phases enters a nonlinear Kerr medium and produce diffraction-limited beam through the self-focusing (collapse) of multiple laser beams into the single output beam. (b) Cross-section of these beams before entering medium for 3 and 7 beams.



Figure 2: Breaking of water waves.