Math 1220 Final Review

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Graphing

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Library of functions and Transformations

Know how to graph ALL of the functions in the library of functions

$$f(x)=c \quad f(x)=\sqrt{x} \quad f(x)=e^x \quad f(x)=\ln x$$

$$f(x)=x \quad f(x)=\sqrt[3]{x} \quad f(x)=a^x \quad f(x)=\log_a x$$

$$f(x)=x^2 \quad f(x)=\frac{1}{x} \quad f(x)=2^x \quad f(x)=\log_2 x$$

$$f(x)=x^3 \qquad\qquad f(x)=10^x \quad f(x)=\log x$$

Know what the domain and range of each of these functions are.
Know how to find intercepts of each of these functions.
Know how to graph transformations of these functions.

Graph the following
State their domains, range, intercepts (if any), asymptotes (if any)

$$f(x)=\sqrt{x-3}+2$$

$$f(x)=-e^x+2$$

$$f(x)=-\log_3(x-1)$$

$$f(x)=2^{-x}-8$$

Linear Functions

Find the slope, the _y_ intercept
Parallel slopes, perpendicular slopes
Equations of horizontal lines, vertical lines

$$f(x)=mx+b$$

Given the line $4x-5y=7$.
- Find the slope of a line that is parallel to this line.
- Find the slope of a line that is perpendicular to this line.

Piecewise Functions

Graph the following
State the domain, range

$$f(x)=\begin{cases} x^2+3 & x\le 0 \\ -2x+1 & x>0 \end{cases}$$

Quadratic Functions

Graph the following. State the domain, range, the vertex, the axis of symmetry, any intercepts

$$f(x) = -(x-2)^2 + 4$$

Find the vertex of the function

$$f(x) = 2x^2 - 6x - 8$$

Complete the square and write in the form $f(x) = (x-h)^2 + k$

$$f(x) = x^2 - 6x + 1$$

Find the intercepts of the quadratic function $f(x) = x^2 - 3x + 2$

Find the zeros of the function $f(x) = x^2 + 2x - 11$

Find the equation of the quadratic function with vertex at $(1, -5)$ and that goes through the point $(0, -3)$

Polynomial Functions

Graph the following
State the degree, end behavior, y intercept, zeroes and their multiplicities

$$f(x) = x^2(x-3)(x+1)^2$$

Notation for end behavior

$$x \to \infty, \quad f(x) \to ?$$
$$x \to -\infty, \quad f(x) \to ?$$

Rational Functions

State the domain, intercepts, any asymptotes and graph

$$f(x) = \frac{(x+3)(x-1)}{(x-3)(x+1)}$$

State the domain, vertical and horizontal asymptotes, intercepts and graph

$$f(x) = \frac{1}{x-3} + 2$$

Circles

Graph the following
Find the intercepts

$$x^2 + (y-3)^2 = 16$$

Find the equation of the circle in standard form that has center (-3,5) and radius 7.

Functions
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Function Notation
Average Rate of Change

Find the Average Rate of Change

$f(x) = x^2 + 2x - 3$ from $x = -1$ to $x = 3$.

$f(x) = \dfrac{3x}{x+1}$ from $x = -2$ to $x = 2$.

Difference Quotient

Find the Difference Quotient $\dfrac{f(x+h) - f(x)}{h}$

$f(x) = 4x - x^2$

$f(x) = \dfrac{4}{x}$

Even/Odd

Is $f(x) = \dfrac{3x}{x+1}$ even/odd/neither?

Domain and Range

Find the domain. Give the domain in interval notation.

$f(x) = \dfrac{1}{\sqrt{2x+3}}$

$f(x) = \dfrac{2x+1}{x^2 - 4}$

$f(x) = 7\ln(2x - 5) + 3$

Inverses

Find the inverse of the following functions
State the domain and range of both the function and its inverse

$f(x) = \dfrac{x+2}{x-3}$

$f(x) = \sqrt{x+1} + 2$

$f(x) = \dfrac{3x}{x+8}$

$f(x) = \log_3(x-1) + 2$

$f(x) = e^{2x-3}$

Composition

For each pair below, find the compositions $(f \circ g)(x), \ (g \circ f)(x), \ (f \circ f)(x)$
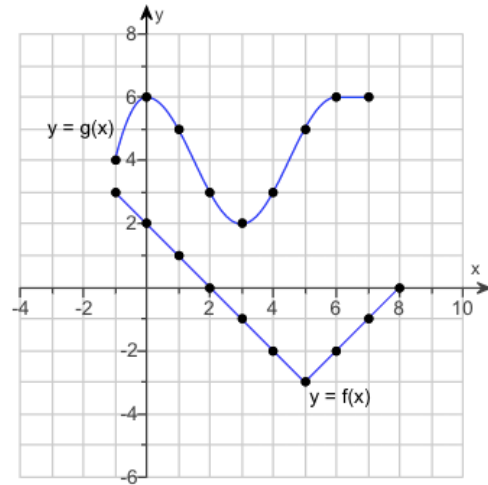
State the domain of the composition

$$f(x) = 1 - 2x, \quad g(x) = \sqrt{x - 2}$$

$$f(x) = \frac{x - 5}{x + 1}, \quad g(x) = \frac{x + 2}{x - 3}$$

$$f(x) = 2x^2 + 7, \quad g(x) = \sqrt{x - 3}$$

Given the graphs of f and g find $(f \circ g)(4)$

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Techniques and Applications

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Solving Equations and Inequalities

Solve the following. Write your solution in interval notation.

$$|3x - 7| \geq 1$$

$$|2x + 3| < 7$$

$$3x^2 \geq 14x + 5$$

$$\frac{3x - 5}{x + 2} \leq 2$$

$$(x - 1)(x - 2)(x - 3) \leq 0$$

Completing the Square

Write the following equation of a circle in standard form

$$x^2 + y^2 - 6x + 2y + 4 = 0$$

Distance Between Points, Midpoint of line segment between points

Find the distance between the two points given.

Find the midpoint of the line segment connecting the points.

If the two points are ends of a diameter of a circle, what is the equation of the circle? $(2, -5), (4, 7)$

Solving a system of equations

Solve the following system of equations

$$\begin{cases} 2x + 3y = 1 \\ -x + y = -3 \end{cases}$$

Exponent and Logarithm Rules

Simplify the following using rules of exponents and logarithms

$\ln\left(e^3\right)$

$e^{3\ln(2)}$

$2\log(100)$

$10^{\log(3)}$

$\log_2 32 - 2\log_2 4$

Solving logarithmic and exponential equations

Solve the following equations

$36 - 4e^{2x+1} = 20$

$8^{x-1} = 5^{2x+1}$

$2\ln(x-7) = 1$

$4e^{-3x} = 20$

$9^{15-x} = 27^x$

$\log_x 64 = 2$

$4^x - 2^x - 12 = 0$

$\log_2(x+3) + \log_2(x-1) = 3$

Applications

- Find the amount in your account if you invest $2000 at an interest rate of 3% compounded continuously. How long will it take to double your investment?
- Find the interest rate if you invest $3000 in an account compounded continuously and have $5000 in the account after 8 years.
- A substance decays at a rate modeled by $A = A_0 e^{-0.04t}$, where A is in grams and t is in years. If there are initially 5 grams of the substance, how much will there be after 8 years? How long will it take for there to be only 1 gram of the substance remaining? What is the half life of the substance?
- A projectile is launched vertically upward and its height (in feet) at time t (in seconds) is described by $h(t) = -16t^2 + 32t + 24$. When does the projectile reach its maximum height? What is the maximum height of the projectile? At what height was the projectile when it was launched? When does the projectile hit the ground?
- The weekly rental cost of a 20 foot recreational vehicle is $129.50 plus $0.15 per mile. Find a linear model that expresses the cost C as a function of miles driven, m. What is the rental cost if 860 miles are driven? How many miles were driven if the rental cost is $213.80?
- The price p in dollars and the quantity sold, x, of a certain product obey the equation $p = -\frac{1}{10}x + 1000$. Find a model that expresses the revenue, R, as a function of x. What is the revenue if 400 units are sold? What quantity x maximizes the revenue? What is the maximum revenue?
- A farmer with 3000 feet of fencing wants to enclose a rectangular field. Express the area A of the rectangle as a function of x where x is the length of the rectangle. For what value of x is the area largest? What is the maximum area?
- A movie theater sells tickets for $10.00 each, with seniors receiving a discount of $2.00. One evening the theater sold 525 tickets and took in $4630 in revenue. Set up a system of equations for this situation. Be sure to state what each variable stands for. How many of each type of ticket were sold?