Independent Thinking

Reuben Hersh

Reuben Hersh (rhersh@math.unm.edu) was born in 1927. He received a Ph.D. degree in Mathematics from New York University in 1962. He is now living in Santa Fe, New Mexico, where he no longer teaches part time at Santa Fe Preparatory School.

Lisa had asked me to do an admissions interview. The twins were applying for the seventh grade. The Academy doesn't usually like to take home-schoolers without being really sure they are ready for a more conventional setting. Ludwig and Imre had each scored 99.5% on arithmetic, but there was one kind of funny response.

The two boys looked like the victims of a strict upbringing. They were wearing jackets and ties, and respectful expressions.

R: Why do you want to come here to the Academy?

Ludwig: Our mother thinks it's time we learn to mingle.

Imre: It's supposed to be better than public school.

R: OK, and how do you feel about it yourselves?

Imre: Fine.

Ludwig: Sure. It's OK.

R: All right. Do you like math?

Imre: It's fine.

Ludwig: The test was easy.

R: You'll find that math gets harder as you go along.

No response.

R: You both missed the same question on the test. Do you remember this one: 2, 4, 8, 16, ?

Ludwig: Yes, I remember that. It was easy too.

R: You answered, 16.

I: That's wrong. I got the right answer.

R: You answered, 2.

I: Yes, that is the answer.

R: No, you were both wrong. The answer is 32.

I: How do you know?

R: How do I know? I'm the math teacher here!

L: Well, 16 really is the right answer.

R: No, I'll explain it to you. Starting with 2, you double each number. 4 is twice 2. 8 is twice 4. 16 is twice 8. So the next number is 32, because that's 2 times 16.

L: All right. I see how you get that.

R: So now, if we want to continue one step further, what would the next number be?

L: It would be 32.

I: No, no, the answer is still 2!

I was stunned. For a minute, I was tempted to shout. But I didn't. I took a breath, forced a smile and said

R: All right, Ludwig. Why do you think the answer is 32?

L: Because 32 is big enough, you can stop there.

I: You dope! Ludwig, can't you see that you don't just stop, you start over when you come to the end?

R: All right. You each have an explanation, kind of. Ludwig, you think that whatever is the last number you're given is big enough, you can just stop there. Is that right?

L: I don't know. What's wrong with that?

R: And Imre, you think that the last number in the sequence is a signal to go back to the beginning. Is that right?

I: Well, you can't just stay in the same place forever, can you?

R: But what's wrong with going on doubling each time?

L: That's OK, you can do that if you want to.

I: Sure, no problem.

R: Thank you. I'm glad you admit that it's OK to say 64 is the next number.

L: Why not?

R: Let me ask you this. You realize, don't you, that you can always go on doubling as far as you like?

I: You can? How do you know?

L: It seems that after a while you would get tired and give up.

R: Well, that's true, I guess. I mean, in principle.

L: What's the principle?

I: Yes, teach us your principle.

R: That's the principle. You can always go on.

L: You mean, you can always go on because you can always go on.

R: Are you trying to be funny?

I: No, he never tries to be funny, Mr. Hersh.

R: Good. I want you to think independently, but don't try to be funny.

No response.

R: Forget about doubling. Can you count?

I: Sure. 1, 2, 3, 4, 5, 6, 7...

R: OK. Very good. Now you see that there's no end to counting, don't you? You can always go on? You can always add one more?

L: Well, what do you mean, always?

R: It doesn't matter what I mean, always. You can just always add one more!

I: Did always ever happen yet?

R: You're getting smart again.

No response.

R: Look, everyone knows you can always add one more. It's obvious. How come you didn't learn that at home, long ago?

L: We never talked about always. We could ask our mother about it tonight.

I: No, she would just say, decide for yourself. It's up to you.

R: Well, that's good. You should learn to think for yourself. Don't take anybody's word for things. Be independent and critical.

L: All right.

R: So look, you know decimal notation and place value, I can see that from your admission test.

No response.

R: Right? You know you can just add a zero at the end, that's the same as multiplying by 10? And you know how to add, so you can always add a 1 to any number?

I: Yes, we know that. It's easy.

R: Well, then, you must see that you can always go on, you can always add 1, or even multiply by 10.

L: Well, that's all right if you say so.

R: No, no not because I say so! Think for yourself! Can't you see that it is so?

I: If I think for myself, I think that eventually you have to start over.

L: No, you don't. Eventually, you get tired, or die, or run out of paper.

R: Well, Ludwig, what you say is true, but you're just not getting the point. Getting tired or dying or running out of paper isn't math. It's biology or sociology or whatever you want to call it. We're doing math here.

L: Does that mean math says you can always go on because that's what math says?

R: Right! Right! You're getting it, finally.

I: Where is that written? Is it in some book?

R: No, it's not in any book. It doesn't have to be in any book, because everybody knows it. Only because you haven't been in regular school, you are just finding out about it now.

L: What if we asked some other math teacher? Would they say the same thing?

R: Absolutely. Every math teach in the world will say the same thing.

I: How do you know?

R: Because otherwise they wouldn't be allowed to teach math.

No response.

R: So, let's go back to the beginning. What's the next number? 2, 4, 8, 16, 32, 64 . . .

No response.

R: If you want to get into the Academy, you better answer.

L: Maybe I'd like public school better.

I: I don't know. This is harder than I thought it would be.

R: Come on, you know the answer is 128.

L: I know that's what you want me to say.

I: Yes, that's true. We have to know what you want us to say, and say that.

R: No, you still don't understand. It's not what I want. It's really the right answer. You know 128 is really the right answer.

I: The right answer is the answer you want.

R: Well of course I want you to give the right answer! I'm the teacher!

No response.

R: Well, we'll let your mother know about admission to the Academy.

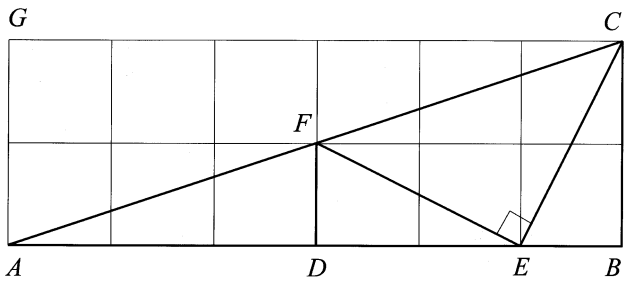
I was really glad I had the chance to interview them. I sure don't want them in any class of mine. In public school, they'll just be told to say what they're told to say. At the Academy, we insist on independent thinking.

Arctangent Identities

Rex H. Wu (Brooklyn, New York, RexHWu@aol.com) says that the figure contains all the identities

$$\tan^{-1}(1/2) + \tan^{-1}(1/3) = \pi/4$$
$$\tan^{-1}(3) - \tan^{-1}(1/2) = \pi/4$$
$$\tan^{-1}(2) - \tan^{-1}(1/3) = \pi/4$$
$$\tan^{-1}(1) + \tan^{-1}(1/2) + \tan^{-1}(1/3) = \pi/2$$
$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$$

and challenges readers to find them.



A diagram with labels that may help appears on page 138.