# Mathematical Menopause, or, A Young Man's Game?

*m y mathematical career was nonstandard. I started graduate school at age 29. From 34 to 50, I produced research, much of it well received. After 50 I could no longer create new mathematics. I popularized and philosophized from then until now, age 72. How* typical or how strange has my story been?

Great authorities warn us, "Mathematics is a young man's game" (see later, under "Hardy vs. Littlewood"). My starting age for research contradicts the rule, but my concluding age seems to verify it.

Albert Einstein said, "A person who has not made his great contribution to science before the age of thirty will never do so." [6]

The French-Jewish number theoretician André Weil wrote, "Mathematical talent usually shows itself at an early age. There are examples to show that in mathematics an old person can do useful work, even inspired work; but they are rare and each case fills us with wonder and admiration." [13] The Bourbaki collective expels members at 50.

At his fiftieth birthday party the great German function-theorist Felix Klein whispered to his English student Grace Chisholm Young, said to be his favorite pupil, "Ah, I envy you. You are in the happy age of productivity. When everyone begins to speak well of you, you are on the downward road." [14]

In a *New Yorker* article, "Mathematics and Creativity," Alfred Adler wrote:

... consuming commitment can rarely be continued into middle and old age, and mathematicians, after a time, do minor work. In addition, mathematics is continually generating new concepts which seem profound to the older men and must be painstakingly studied and learned. The young mathematicians absorb these concepts in their university studies and find them simple. What is agonizingly difficult for their teachers appears only natural to them. The students begin where the teachers have stopped, the teachers become scholarly observers. [1]

On the other hand, in his biography of the Israeli-American logician Abraham Robinson [5], Joseph Dauben wrote,

[Robinson] was always pleased to dispel the myth that the best mathematicians were under thirty and that a mathematician did her or his best work early, at the very start of one's career. As a striking counterexample, Robinson's best mathematics was only beginning to reap the benefits of his wide experience when, suddenly at the age of fifty-five, he died.

Abraham De Moivre (1667–1754) found his presumably most important result when he was 66—the "local central limit theorem." . . . De Moivre had to stay competitive as a problem-solver in order to attract noble coffee house frequenters as paying clients for instruction. De Moivre in old age used to sleep every day a bit longer until the sleeping phase reached 24 hours. (Ivo Schneider, e-mail communication)

Weierstrass was 70 when he discovered polynomial approximation.

The English-Jewish algebraist J. J. Sylvester pointed out that Leibniz, Newton, Euler, Lagrange, Laplace, Gauss, Plato, Archimedes, and Pythagoras all were productive until their seventies or eighties. And of course, we would add Sylvester himself. "The mathematician lives long and lives young," he wrote. "The wings of the soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life."

"In 1896, in the eighty-second year of his age, Sylvester found a new enthusiasm and blazed up again over the theory of compound partitions and Goldbach's conjecture." (Bell, p. 405) (We would omit Plato and Pythagoras; and G. H. Hardy tells a different story about Newton.)

I decided to do my own check on Hardy. Is mathematical aging so inexorable? I mailed out a questionnaire. Since Hardy's apology was in large part a self-evaluation, it seemed fair to base my research on self-evaluations.

From the American Mathematical Society membership directory, I chose 250 names. I had known most of them somewhere, at some time. They were mostly Americans, with a sprinkling of American women. There were a few Canadian, Swedish, French, Israeli, and Japanese mathematicians I knew who had spent time in the United States.

Reflecting my training and experience, the mailing list was heavy on differential equators. Theoretical d.e.'s (both o. and p. and s. as well). Applied d.e.'s. Numerical d.e.'s. There were also stochastic processors, and a scattering of logicians, algebraists, topologists, geometers, and statisticians. I see no reason why mathematical specialty makes much difference for these questions.

I got 66 replies, which is said to be a very good response rate. They came from 23 states, plus Ontario, Alberta, British Columbia, Sweden, and Israel. California and New York led with 11 and 9 respondents, respectively. Ages ranged from 54 to $92\frac{1}{2}$. 47 were over 60 years old, and 22 were over 70.

Some of my old acquaintances were happy to get back in touch. There are many names a reader would recognize. I also include a quotation from Ivan Niven's interview in the *College Mathematics Journal*.

## The Questionnaire

Here are the questions I sent:

1. What opinions or information do you have about aging and mathematicians?
2. How old are you now?
3. How old were you when you started mathematical research?
4. What have been your main fields of mathematical research? How would you compare the value and interest of your research at the beginning of your career and that of your most recent research? How do you think the mathematical community compares them?
5. Did you find at a certain age that you had lost some zest or drive or facility for mathematical research? At what age? What happened?
6. Did you have such experiences more than once?
7. Do you attribute them to aging or other causes?
8. Did you give up your research work? Did you switch to another field of research? Which one? With what success? Did you go from pure to applied? Theoretical to numerical? Was the new field close to the old one, or much different?

9. Did you then develop a more intense and committed interest in teaching mathematics? In writing treatises and textbooks?
10. Did you collaborate more or less? Did you collaborate with juniors, equals, or seniors? (G.-C. Rota: "At my age the work of the collaborator is crucial.")
11. Did you develop a new serious commitment to non-mathematical activities? Which ones? Have those activities been able to replace mathematical research for you?
12. In your mature years have you tended to return to the subjects and problems of your youth? (Hille)
13. Did you feel a strong sense of loss in giving up your earlier research goals?
14. Did these experiences affect your standing in your department? How? By smaller pay raises? By less influence in decision-making in your department? In the math profession?
15. Have you suggestions for individuals or institutions to prolong the period of active research?
16. Other questions, comments, suggestions?

# Hardy vs. Littlewood

G. H. Hardy famously opined, "Mathematics is a young man's game." In *A Mathematician's Apology*, he explains: "I write about mathematics because, like any other mathematician who has passed sixty, I have no longer the freshness of mind, the energy, or the patience to carry on effectively with my proper job . . . If then I find myself writing not mathematics but 'about' mathematics, it is a confession of weakness for which I might rightly be scorned or pitied by younger and more vigorous mathematicians. . . . I do not know an instance of a major mathematical advance initiated by a man past fifty. If a man of mature age loses interest in and abandons mathematics, the loss is not likely to be very serious either for mathematics or for himself. . . . Mathematics is not a contemplative but a creative subject; no one can draw much consolation from it when he has lost the power or the desire to create; and that is apt to happen to a mathematician rather soon. It is a pity, but in that case he does not matter a great deal anyhow, and it would be silly to bother about him. . . . A mathematician may still be competent enough at sixty, but it is useless to expect him to have original ideas."

Hardy had two great collaborators, Ramanujan and Littlewood. Ramanujan died at thirty-three. What about Littlewood?

In 1941, when Hardy wrote his *Apology*, Littlewood was already 56. From the introduction to *Littlewood's Miscellany*, by Bela Bollobas: "In 1950, at the statutory age of 65, Littlewood retired and became an Emeritus Professor. The Faculty Board realized that it would be madness to lose the services of the most eminent mathematician in England, so they wrote to the General Board: 'Professor Littlewood is not only exceptionally eminent, but is still at the height of his powers. The loss of his

teaching would be irreparable, and it is avoidable. Permission is requested to pay a fee of the order of 100 pounds for each term's course of lectures.' The response: 15 pounds per term, the fee paid to an apprentice giving his first course as a try out, to a class of 2 or 3. So Littlewood gave courses at 15 pounds for 4 years. He tried to stop once but there was a cry of distress. At the same time he turned down lucrative offers from the United States . . . Littlewood remained active in mathematics even at an advanced age: his last paper was published in 1972, when he was 87. One of his most intricate papers, concerning Van der Pol's equation and its generalizations, was written when he was over seventy: 110 pages of hard analysis, based on his joint work with Mary Cartwright. He called the paper 'The Monster' and he himself said of it: 'It is very heavy going and I should never have read it had I not written it myself.' His last hard paper, breaking new ground, was published in the first issue of the *Advances in Applied Probability*, when he was 84. . . . In 1972 Littlewood had two bad falls and he fell again in January 1975. He was taken to the Evelyn Nursing Home in Cambridge, but he had very little interest in life. In my desperation I suggested the problem of determining the best constant in Burkholder's weak $L_2$ inequality (an extension of an inequality Littlewood had worked on). To my immense relief (and amazement!) Littlewood became interested in the problem. He had never heard of martingales but he was keen to learn about them. . . . All this at the age of 89 and in bad health! It seemed that mathematics did help to revive his spirits and he could leave the nursing home a few weeks later. From then on, Littlewood kept up his interest in the weak inequality and worked hard to find suitable constructions to complement an improved upper bound."

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One recipient thought my questionnaire was biased toward pessimism—reflecting my own depressed personality. Many respondents praised my project. They called it "refreshing," "provocative," "most worthy." One wrote, "I wish, as probably many people do, that there was something with a little authority written, and I am very pleased that someone of your stature is undertaking it."

Responses trickled in for six months. It took another six months to absorb them and to see a way to present them. There is no claim that my choice of 250 was "typical," let alone "random." And the 66 of 250 who answered are certainly not typical. They are biased toward people who answer questionnaires, who like to hear from an old acquaintance, who are willing to consider some possibly painful issues, and who aren't too unhappy or ashamed of their lot in mathematical life. The people who don't respond to questionnaires are like the dark matter of the cosmos; we know they are out there, but we can only guess what they look like.

Most responses didn't deal directly with Hardy's claim that if you're ever going to do anything important you must do it when you are young. A differential geometer from California pointed out that this isn't the same question as whether you are still active in old age. (Or, as people say nowadays, when you're "older.")

Two of my respondents knew of earlier surveys by famous mathematicians. One respondent said George Mackey did a study of 50 leading mathematicians, and concluded that on average their best work was done in their late 30s. Another respondent said Gail Young did a study of people who matured very young in mathematics. He found that they generally burn out early. Young felt there was a fairly constant period during which a person could do very creative work. Some had their period earlier, others had it later.

The questionnaire invites recipients to tell as much as they like about their current and past situations. The an-

swers yield a glimpse at how this sample of mathematicians view their lives in mathematics. Such responses don't submit readily to tabulation, much less to statistical analysis. The interest is not only in the consensus but also in the many individual points of view. I wish I could quote all of them.

Most of my respondents are satisfied with their life situation! Relevant is [12], a report by S. S. Taylor on retirees (not restricted as to field) at the universities of New Mexico and Rhode Island in the U.S. and Bath and Sussex in England. Reassuringly, perhaps surprisingly, 98% of the UNM retirees, 97% of the Rhode Island retirees, and 94% of the English retirees told Taylor they are "reasonably satisfied" or even "very satisfied" with retirement. Two-thirds of the American respondents told her they receive the same or higher income as before retirement.

Most of my respondents say they continue research after retirement. Some think their recent research is their best ever. Some say they're doing what they're interested in, uninhibited by the judgment of the math community.

I organized the answers into 7 groups. Excerpts from some answers appear in more than one group.

1. No general statement can be made about mathematical aging.
2. Mathematicians are best in youth.
3. Mathematicians may be as good or better in later years.
4. Symptoms and strategies.
5. Penalties for aging and for following one's own bent.
6. Advice for aging mathematicians.
7. Advice for mathematics departments.

Many responses are hard to classify. For example, "Kato, at about age 75, just published a very good paper, though he does complain bitterly about not being able to do good research any longer." Is that group 2, group 3, or both?

Some painful experiences in group 5 contradict some advice in group 6. Many respondents say, follow your own bent, regardless of outside pressure; and many respondents report penalties for doing that.

Some respondents don't give their age; for a few, I was unable to identify geographical location.

**Group 1: No general statement can be made about mathematical aging.**

"All generalizations are false, especially this one."

"Better not generalize—we are all different." *Probabilist, British Columbia, age 62*

"I met Pólya when he was past 70, and I thought he would go on forever, while I have met a number of promising mathematicians who faded before 30." *Analyst, Maryland, age 79*

"I don't see any general patterns. Some never enjoyed research, some enjoy it but don't want to do a lot of it. On

a superficial view these are indistinguishable from people who lose zest at age 35, but the reason is different; when they achieve tenure their research declines, just because the pressure for it is gone." *Matrix-theorist, Ontario, age 73*

**Group 2: Mathematicians are best in youth.**

Here we hear of some sad, even tragic experiences.

"One of my old, dear friends suddenly went dry in research at age 40. It was very traumatic for him, and for me to observe." *Analyst–applied mathematician, Texas, age 54*

"My zest is fine, but capacity much diminished before age 55. Age and alcohol and depression." *Analyst, California, age 72*

"One does best between age 20 and 50. My most recent research (c. 1996) is not as good as my work in the 1950s." *Differential geometer, California, age 67*

"At around 55 I had lost whatever originality I once possessed. But not the desire to learn and *try*." *Analyst, Maryland, age 79*

"I used to work late at night, but now I'm too exhausted to do more than make calendar entries and clean up my study." *Analyst, Louisiana, age almost 62*

"Clearly at my age I can't keep up with the best younger people. Some old-timers have looked foolish in their later research efforts. My hope is at least to avoid that." *Applied mathematician, Rhode Island, age 71*

"The vast majority of mathematicians do best before 40, and often as not, before 30. But that would be hard to substantiate; one would need to know the life work of a mathematician, and make reliable judgments about it. . . . The best counterexample I know is Legendre, who proved the case n = 5 of Fermat's last theorem when he was in his seventies." *Differential geometer, California, age 73*

"Men age faster than we girls. It makes up for them being bullies earlier. How to pep them up? I try . . . People whose lives are fairly stable and satisfactory keep going a lot better. One of my colleagues gave up research at 42 when his marriage broke up. Another similar at 48." *Probabilist, British Columbia, age 62*

"As you get older you know too much. You have all these methods, and you try all the combinations and variations you can think of. You're running down the old tracks and nothing works." (Ivan Niven, [2])

"Mathematics tends to be introverted, with unbalanced expenditure of mental energy. As one grows older there is desire for other forms of expression, which dilutes the intensity to solve problems. 'What does it all mean?' is asked more often, which also can slow down progress."

"Aging has two sides—your own age, and the age and aging of your subject and your contributions. This aging is brought about by the work of younger competitors." *Analyst, Sweden, age 80*

## "As you get older you know too much."

"The field of mathematics moves very fast. The pace has been quite extraordinary in the past 50 years. Just trying to keep up in one's specialty requires many hours of effort. One doesn't feel comfortable doing the same old thing. Some great mathematicians have been unable to handle this. When a decent problem comes along which seems accessible, I'm eager to jump in. The trouble is that the frontier is moving so fast. It's not that we give up research mathematics, research mathematics gets away from us." *Geometer, California*

**Group 3: Mathematicians may be as good or better in their later years.**

"A Young Woman's Game?", below, provides impressive testimony that women mathematicians are often at their best in their 30s, 40s, even 50s.

"Mordell is supposed to have said modestly, 'I did work in my 70s many a younger man would have been proud to have done.' Among my teachers, I know that Beurling, Ahlfors, Zariski, Mackey worked intensely on research when they were quite old." *Analyst, Rhode Island, age 72*

"Since I became emeritus in July 1995 my research has increased. Most of it is joint with former students and post-docs. The mathematical tools are ones I've used before—this is probably typical. It is a great relief to shed 9 years of department chairmanship, too many committees, and obligations to seek external grants. I no longer attend department meetings." *Analyst–applied mathematician, Rhode Island, age 71*

"Some of my best work was done after age 47. Possible motivations were a bad spell of drinking and divorce from 1974 to 1977, and prostate cancer treated successfully by radiation and implants. After such trauma I tend to over-accomplish." *Analyst, Illinois, age 69*

"Knowledge and experience count for a lot more than CPU speed. At the minimum it improves your mathematical taste. My recent papers are a lot better than those just after the Ph.D." *Analyst, Alabama, age 61*

"Young guys may luck out but often only when someone older points the way." *Applied mathematician, Colorado, age 64*

"The young may find gold but cannot read the land; the older have familiarity with the landscape, which guides them to where to dig."

"Recently a friend compared me with Brahms, who turned out great works throughout his life! I hope to live up to the praise." *Numerical analyst, Ohio, age 70*

**Group 4: Symptoms and strategies.**

By this heading I mean symptoms of aging, and strategies to cope with it.

"My wife and I have been happily married for forty-four years; that's extremely important. Our garden takes a major part of our time in the growing season." *Applied mathematician, Rhode Island, age 71*

"My memory is not what it used to be. My work takes much longer and the need for careful notes is greater. My best work was around age 40." *Analyst, Sweden, age 68*

"My drive for research and my direction haven't changed dramatically. I just don't think as clearly and quickly now. But I've grown efficient in other respects, and as a result my best work has been in the last 15 years. I collaborate more, but still do a lot by myself." *Applied mathematician, Utah*

"As I age my memory declines, making it more difficult to keep in mind all the threads of a complicated situation. Also my computational abilities decline—I take longer to get through a routine calculation, and make more mistakes.

## A Young Woman's Game?

Does the slogan "mathematics is a young man's game" exclude women? Or should it be "a young person's game"? Claudia Henrion [8] says no. " . . . there is a deeply entrenched belief that mathematics is a 'young man's game,' despite the fact that there is no compelling evidence to support this hypothesis; indeed, the studies that have been done suggest the contrary. But when the image and reality differ, it is often the image that can have a more powerful influence on attitudes, practices, and policies. If the focus were not so much on the young, virile mathematician, it would be easier to design programs with women in mind. For example, [recognizing] the fact that women are likely to have children in what is traditionally considered their prime mathematical years . . . looking at their productivity over a longer time span . . . recognizing that women may need to enter the mathematical research pipeline later in life, as Joan Birman did, or they may need to work part time for a period to balance having children with mathematical research, as Mary Ellen Rudin did."

The prominent logician Marian Pour-El told Henrion, "I've never felt that you're over the hill if you're in your late thirties. I think I've done my best work later on, by a long shot."

The leading braid-theorist Joan Birman focused better on math after the issues of marriage were settled, her children were older, etc. "I think doing mathematics when you're enthusiastic is important—not your age."

Rudin, a famous topologist, said, "I don't think most people's best work will be done by the time they're thirty, and certainly my best work wasn't done until I was fifty-five years old."

I catch mistakes by my sense of what seems right, rather than by repeating computations. On the other hand, I'm more canny in developing effective research strategy, and more daring in carrying it out. . . . I have an intellectual home with a small but active worldwide community of scholars with similar interests." *Applied mathematician, Age $70\frac{1}{2}$*

"I toy with retiring at the end of this year. I am nervous about it, but clearly recognize the diminishment of my ability to do first-rate research. The main cause is inability to stick with messy detailed manipulations. In the past I could calculate for hours, but now I shy away from such grunge. I still have plenty of things to work on, but I pick them more carefully." *Numerical analyst, California, age 74*

"Getting old is a pain. I still do decent mathematics. However, what I do is very much related to my previous work. I do not jump into a new field, because I have not the same intuition as earlier to 'know' it will lead to something. Everything takes much longer to complete and I make more mistakes, or better, I do not know immediately when the result is wrong. So I have to check much more carefully. I have been a good thesis advisor, which I enjoyed very much. Former students still speak to me, and I still work with them. But I have no students any more, because I cannot be sure I will be around in 4 years. Also, young people should work with young people on 'modern' problems. There can be one advantage with old age. If one is lucky and in balance with oneself, one can look at the world as an independent observer." *Numerical analyst, Sweden/California*

"The principal obstacles to continuing research are: (a) Research requires energy, and this is in increasingly short supply. (b) Research requires keeping up with the literature, and this becomes difficult as one's mental and physical energy declines. (c) Good research requires breadth and flexibility, but the tendency as one ages is to concentrate on a narrow path, dominated by what one has always done, and knows well.

"Collaboration is essential in maintaining research activity. I have tended to collaborate with juniors, since very many of my collaborators have been my students. The younger partner provides energy and awareness of what is currently a 'hot topic'; the older provides perhaps greater familiarity with the history of the topic and a larger battery of available methods." *Topologist, New York, age 76*

"There are many useful things someone with mathematical ability can do. But education and research rewards do not encourage people to branch out and explore. They get stuck in the frontiers of their narrow specialty. The going gets rough when they no longer have the ability or willingness for the concentrated effort to do really complicated technical work. I am still able to do this if I get away for a couple of weeks, but at home commitments to family and work preclude that concentration. It does get harder as you get older, from aging but also from accumulation of other responsibilities and interests." *Logician, Indiana, age 57*

"My enthusiasm for research increased rapidly from 25 to 35 and stayed high for 15 years. During that time I could not imagine not doing research. Then I became interested in research in undergraduate mathematics education (RUME). I tried to work in both fields, but my interest and ideas for work in functional analysis disappeared. There are two possible factors. Interest in RUME may have driven out interest in functional analysis. The other was a feeling that I might do other work as good as my best, but I could not do much better." *Math education researcher, South Carolina, age 64*

"My success and pleasure at research is tied to my ability to travel worldwide and make connections with people from diverse cultures. Politics has been extremely important for me to keep my balance, and this is much easier in Canada than the U.S. I have strong human rights interests related to indigenous peoples in North America and the

## Some Numbers

Claudia Henrion ("A Young Woman's Game?") reports an article by Nancy Stern [11], possibly the only article on aging mathematicians in the research literature (as opposed to the anecdotal literature). Thanks to Judith Grabiner for this reference.

Stern's mentor Stephen Cole [4] studied chemists, geologists, physicists, psychologists, and sociologists, and found, "There are basically no differences in the quality of work published by scientists between the ages of 30 and 50. Scientists over the age of 50 are slightly less likely to publish high quality research."

Stern extended Cole's work to mathematicians. She counted publications of 435 "randomly chosen" mathematicians at Ph.D.-granting institutions, and sorted them by age. Since citations of a paper should roughly measure mathematicians' interest in it, she also counted their citations. The numbers in parentheses in the table that follows are the number of sampled mathematicians in each age group.

| Age | Mean number of publications | Mean number of citations of single-authored and first-authored work |
|---|---|---|
| < 35 | 5.12 | 2.73 (101) |
| 35–39 | 7.33 | 3.80 (96) |
| 40–44 | 6.24 | 5.79 (67) |
| 45–49 | 3.49 | 3.44 (63) |
| 50–59 | 5.22 | 5.63 (73) |
| 60+ | 6.11 | 5.09 (35) |
| TOTAL | 5.64 | 4.22 (435) |

Stern concludes: "The claim that younger mathematicians (whether for physiological or sociological reasons) are more apt to create important work is unsubstantiated. . . . I have found no clear relationship between age and achievement in mathematics."

Third World. The extent to which these activities replaced mathematics interests, or just lived actively beside them, is extremely difficult for me to say." *Numerical analyst, British Columbia, age 54*

## Group 5: Penalties for aging or for following one's path

There are really two topics here. Penalties of either type present an opportunity for some soul searching, by mathematics departments and organizations.

"As I get closer to retirement I am consulted less about departmental matters." *Numerical analyst, California, age 74*

"For sure I have less influence in the department, and get (and deserve) tiny raises." *Analyst, Louisiana, age almost 62*

"Smaller pay raises. Less influence. No NSF grants. The *mafias* protect themselves. I'm treated like a half-breed in the old West." *Analyst–applied mathematician, Colorado, age 64*

"My recent work is more interesting and valuable. Math community isn't interested. Ecology community is." *Applied mathematician, British Columbia, age 65*

"I attached equal value and interest to all my research. The mathematical community attached little of either to any of it." *Analyst–historian, Washington State, age 73*

> "My research is worth more to me than to the department, . . ."

"I have been treated well. I still have my office, 10 years after retirement." *Measure theorist, California, age 80*

"My department has treated me well. I still have an office and they pay me a small amount for looking after some graduate students. My research is worth more to me than to the department, so there is no strong reason why they should actively support it." *Numerical analyst, Sweden, age 66*

"By following my bliss, I gave up my opportunity to get to full professor. My most valuable professional achievements are not appreciated by the leadership in my department." *Age 62*

"It was ignominious that the Fields committee turned down Wiles's achievement on account of his age. There is no formal constraint on age for the medal."

"I use knowledge and experience from early areas in learning new things. Shifting fields this way causes one to lose NSF support. This can be regained after a few years, but I got sick of being funded in new 'lives' and stopped applying. The judgment of proposals is silly; many people simply propose what they have already done and try to guess what directions are politically correct. . . . I found many colleagues singularly narrow in focus and rigid in their approach to scientific discovery. It was often difficult to get anyone to look at my papers." *Analyst, Illinois, age 69*

"At 38 I had built enough research reputation to do some writing without serious loss of status in the research community. . . . At times I believe the only criterion department chairmen go by is bringing in contracts and grants. At times I think my colleagues regard me as an eccentric anomaly pursuing non-standard paths." *Analyst–author, Rhode Island, age 77*

"The mathematical community lost interest in my work when fashions changed and I didn't. After a period as chairman when I was 40, I lost influence in the department." *Analyst, California, age 72*

"My best paper was never referred to in the later literature. I tell myself this shows it said the last word on its subject." *Number-theorist, Minnesota, age 62*

"My chief pain has been realizing that my creativity was very limited—I could not conceivably become a significant mathematician. My seniors had higher expectations for me than were finally justified. I have always had an intense passion for music, and this comforted me." *Analyst, Maryland, age 79*

"I did feel a loss when what I was doing was not valued by the mathematical community. It took a while for me to value it for its own sake." *Logician, Indiana, age 57*

"The value of my research = quite high, the interest by others = quite low. The math community doesn't pay attention to most mathematician's work. . . . I am called on a lot to do diplomatic or administrative jobs. I am not a very able administrator, but compared to the great majority of mathematicians, I am an administrative genius." *Applied mathematician, Alberta, age 60*

"Some of my best research has been in recent years, yet I have been getting smaller pay raises and have less influence in my department. The situation of some of my contemporaries is even more egregious. Mathematics departments and organizations don't pay attention to the older members of the profession. My department treats our retirees shabbily: we give them a 'gold watch' when they retire, then forget about them."

## Group 6. Advice to aging mathematicians.

Recommended by many:
A. Stay in good mental and physical health. Exercise. Don't smoke.
B. Do what interests you, not what others expect.
C. Look for new ideas and areas that appeal to you.
D. Stay away from administration.
E. Don't stop—keep working.
F. Collaborate with your juniors.
G. Have fun.

Here are some quotes:
"First and foremost, you need a deep love of the subject." *Analyst, Alberta, age 60*

"It is important to have an office." *Numerical analyst, Sweden, age 66*

"Stay away from administration. It eats away your creativity, and is a real plague." *Applied mathematician, Utah.*

"Keep working, do not hide behind administrative duties." *Analyst, Sweden, age 68*

Quote from I. Singer: "Keep the pencil moving."

"Work hard, and have several problems to work on." *Logician, age 67*

"Don't stop. Once you do, it's hard to get back. It's not just the field that changes but *you* change." *Geometer, New York, age 65*

"Maintain contact with younger colleagues and students. . . . Whenever anyone asks you a mathematical question, devote at least 15 minutes to it, even if it is 'not my field.' . . . Try to maintain high ethical standards in this competitive profession. I like to think of mathematics as a collective enterprise. We contribute even by being attentive spectators and consumers of the constant outpouring of new ideas. (In the opposite view, a career in mathematics is an ego trip like downhill skiing, reserved for the youngest and strongest, where only those who break records matter.)" *Harmonic analyst, Sweden, age 71*

"Publish less. Communicate more, to better chosen readers. Alberto Calderón told me that important results are found by graduate students because they consider problems afresh. His advice for middle-aged mathematicians was, take up a new problem area; he cited Menahem Schiffer as a model. But he did not follow his own prescription, and he continued to find important results anyway." *Matrix-theorist, Ontario, age 73*

"Hardy advised people to do research in a prone position, so that more blood flows to the head." *Probabilist, Illinois, age 59*

"My early interest in 'useless things' like algebraic geometry paid off handsomely later in soliton theory and strings." *Analyst, Illinois, age 69*

"Always remember, research should be fun. If it becomes too competitive and loses its pleasure, give it up. Don't take your research or yourself too seriously! I have been blessed with a good sense of humour, but how could one suggest this to others?" *Numerical analyst, British Columbia, age 54*

"I do what I can do, and enjoy every minute of it. The mathematical community has as little awareness of me as I of them. Constantly learn new things! Do mathematics just for the fun of it!" *Number-theorist, New York, age 77*

"Stop growing older. Keep having fun. And have a beer, on me!" *Analyst, California, age 83+*

### Group 7: Advice for mathematics departments.

I would like to hope that the following suggestions will be seriously pondered in departmental executive committees.

"Move administrators' pay down. Limit chairs' and deans' terms to 1 or 2 years, as in Japan and Europe." *Applied mathematician, Colorado, age 64*

"I have believed for a long time that a lifetime appointment to research in mathematics, with incidental teaching, is a mistake. A person's abilities, skills, and interests change. I've often talked about a career path involving research at an early age, say 25–35, research and research supervision at a research university next, say 35–45, writing and teaching at a nonresearch university thereafter, perhaps with involvement in preuniversity mathematics such as teacher training or high school teaching. . . . Shorten the time to get a Ph.D. so that people can start research earlier, as in England. Shorten the undergraduate school time for talented people." *Applied mathematician, California, age 74*

"Why not let good searchers search without specifying what they will discover? The AMS establishment spend half their time giving awards to each other, and the selection of referees is capricious or worse. I have been a referee many times and I know that if you expect a proposal to fly you have to say at least 'exceptional' everywhere—'very good' is not enough, and even 'exceptional' may not suffice. This rating system is sublime in its silliness. . . . Students should be screened carefully before their development is undertaken—alternatively, it should be understood that they will sink or swim; it's their problem, not the teacher's." *Analyst, Illinois, age 69*

"Encourage individuals to take chances and follow their true interests." *Applied mathematician, Montana, age $70\frac{1}{2}$*

"I don't think anyone should tell us when to give up." *Analyst, California, age 72*

"Create mechanisms to keep researchers affiliated with strong communities." *Analyst, New York, age 54*

"Do not overburden senior faculty with administrative responsibilities. Encourage them to have research students and to direct Ph.D. dissertations. There is also the complementary problem—senior faculty must recognize when they are no longer effective in research. But they can still provide background and experience, and familiarize younger colleagues and graduate students with methods which have been tried, and difficulties which have been met." *Topologist, New York, age 76*

". . . a good library, some stimulating colleagues and freedom from too many onerous chores. Stan Ulam left USC because teaching calculus to morons was killing him." *Analyst, Alberta, age 60*

"Reward equally. An expert teacher and motivator who designs a great calculus lab is just as valuable as a top-notch researcher." *Combinatorialist, Colorado, age 56*

"For me, a big issue is why retirement is synonymous with severance from all academic activities. The University is the one organism that consciously believes there is nothing to learn from the past." *Number-theorist, New York, age 77*

> ". . . a big issue is why retirement is synonymous with severance from all academic activities."

**My own comments:**

After retirement I asked for a floppy disk, and the person who was then my department administrator told me (even though I was still a part-time instructor), "You're putting me in an embarrassing position. You're not in the budget. You're emeritus now. Why don't you just run over to the book store?"

Shabby treatment of aging professors is not special to mathematics. After retiring from the Columbia economics department William Vickrey got a Nobel Prize for work on transportation economics. A *New York Times* reporter found him in a tiny office far from his department. Vickrey was grateful that after retirement Columbia allowed him any office at all. Perhaps after being written up in the *Times* he would have been granted a better office, but, sadly and unexpectedly, he died a few days later. (See also Littlewood, in Box 1.)

Sometimes emeriti are even dropped from the department e-mail announcements about seminars, hiring, promotions, retirements, and anything else interesting that's going on.

Yet departments can always use extra hands. Undergraduate advising is often understaffed. Has anybody asked, "Are there emeriti who enjoy advising?"

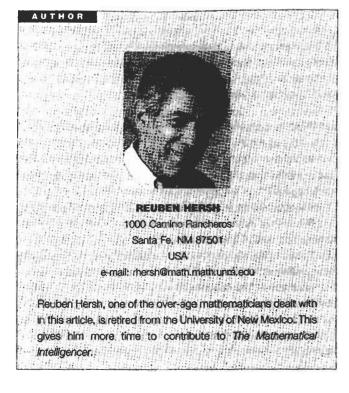If there's no librarian on duty in the math library, is there an emeritus who would serve?

There's always too much committee work. Is there an emeritus with years of service on the undergraduate committee or the master's exam committee? Might he/she have something to contribute there?

**Summary**

The responses were very varied. But these five statements would be generally accepted:

1. There's tremendous variation in how mathematicians age. No one pattern describes everybody.
2. Many mathematicians have been productive in advanced age.
3. To most (not all!) mathematicians, aging brings losses in memory and computing ability. These may be compensated by broader perspective and mature judgment. Possibly more serious is slowness or difficulty in learning new material. Some responses were more specific.
4. Live healthy and follow your own bent, not the pressures of others.
5. Older and retired mathematicians are an under-utilized resource for the mathematics community.

Until we find a consensus about which advances are "major," we can't refute Hardy's claim that no major advance has been made by a mathematician over 50. But his slogan, "Mathematics is a young man's game," is misleading, even harmful. So far as it may discourage people from mathematics when they're no longer young, it's unjustified and destructive.

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Reuben Hersh, one of the over-age mathematicians dealt with in this article, is retired from the University of New Mexico. This gives him more time to contribute to *The Mathematical Intelligencer*.

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