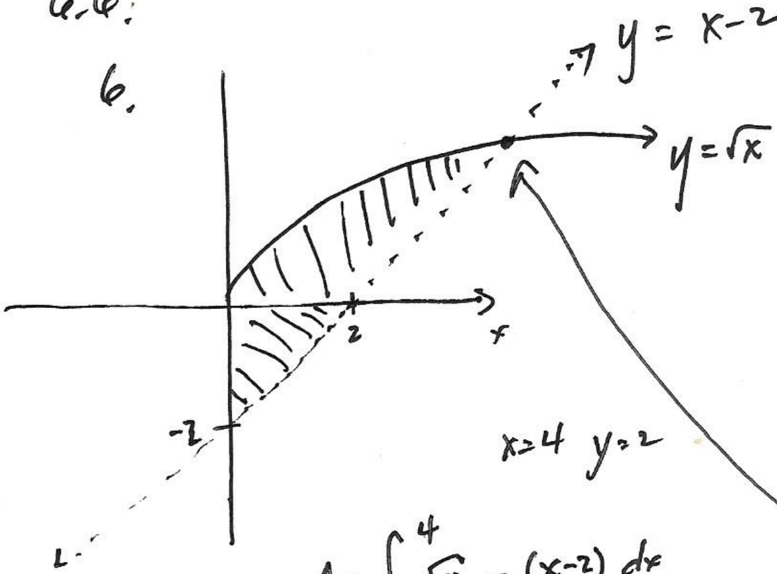
6.6:

6.



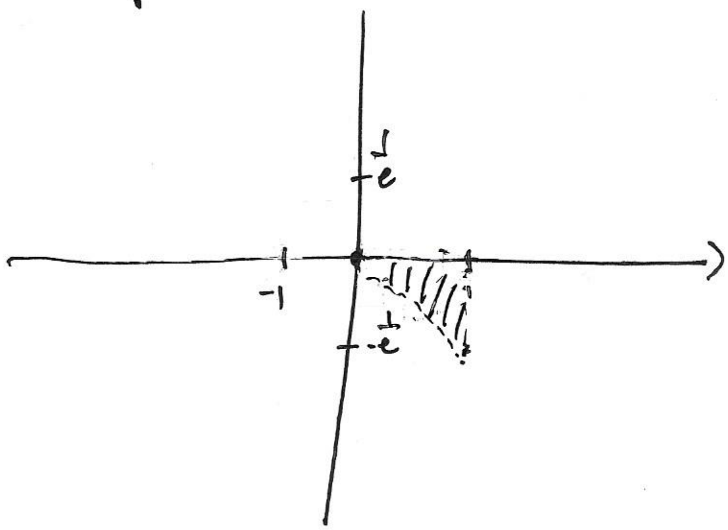
$x=4 \quad y=2$

$$\sqrt{x} = x-2$$
$$x = x^2-4x+4$$
$$0 = x^2-5x+4$$
$$= (x-4)(x-1)$$
$$x=4 \quad \cancel{x=1}, \text{ DNE}$$

$$A = \int_0^4 \sqrt{x} - (x-2)\, dx$$
$$= \int_0^4 \sqrt{x} - x + 2\, dx$$
$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 + 2x \Big|_0^4$$
$$= \frac{2}{3}(8-0) - \frac{1}{2}(16-0) + 2(4-0)$$
$$= \frac{16}{3} - 8 + 8$$
$$= \frac{16}{3}$$

6-6

#16. $f(x) = -xe^{-x^2}$ $a=0$ $b=1$



$y = -xe^{-x^2}$

x	y
0	0
$\frac{1}{2}$	$-\frac{1}{2}e^{-\frac{1}{4}}$
1	$-1e^{-1}$

$$A = \int_0^1 0 + xe^{-x^2}\,dx$$

let $u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

if $x=0$ $u=0$
$x=1$ $u=1$

$$= \int_0^1 xe^{-x^2}\,dx$$

$$= \frac{1}{2}\int_0^1 e^{-u}\,du$$

$$= -\frac{1}{2}e^{-u}\Big|_0^1$$

$$= -\frac{1}{2}\left(e^{-1} - e^0\right)$$

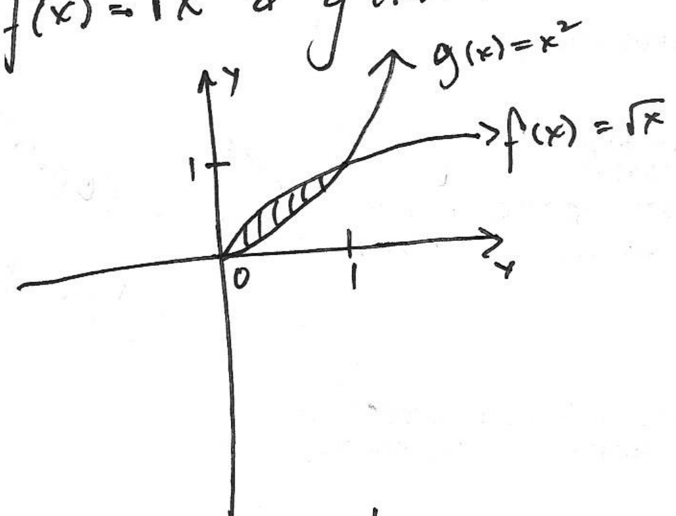
$$= -\frac{1}{2}\left(\frac{1}{e} - 1\right)$$

$$= -\frac{1}{2e} + \frac{1}{2}$$

6.6.

#[illegible]

$f(x) = \sqrt{x}$ & $g(x) = x^2$



$\sqrt{x} = x^2$

$x = x^4$

$0 = x^4 - x$

$= x(x^3-1)$

$= x(x-1)(x^2+x+1)$

$x=0 \quad x=1$

$$A = \int_0^1 \sqrt{x} - (x^2)\, dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3 \Big|_0^1$$

$$= \frac{2}{3}(1-0) - \frac{1}{3}(1-0)$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}$$

46. a) S is the region where Brand A is better than Brand B in removing the smoke from the room.

b) $A = \int_a^b f(t) - g(t)\, dt$

7.1

#8. $\int (x-3)e^{3x}dx$

$= \int xe^{3x} - 3e^{3x}\,dx$

① by parts

$u = x \qquad dv = e^{3x}dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

$= \int xe^{3x}dx - 3\int e^{3x}dx$

① ②

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx - 3\int e^{3x}dx$ ② straight integral

$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} - e^{3x} + C$

$= \frac{1}{3}xe^{3x} - \frac{10}{9}e^{3x} + C$

#20 $\int \frac{\ln x}{x^3}dx$

$u = \ln x \qquad dv = \frac{1}{x^3}dx$

$du = \frac{1}{x}dx \qquad v = -\frac{1}{2}x^{-2}$

$= -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\int \frac{1}{x^3}dx$

$= -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\left(-\frac{1}{2}\right)x^{-2} + C$

$= -\frac{1}{2}\frac{1}{x^2}\ln x - \frac{1}{4}\frac{1}{x^2} + C$

7.1

#32. $\int_1^2 x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2 \ln x \Big|_1^2 - \frac{1}{2}\int_1^2 \frac{1}{x} x^2 dx$

$= \frac{1}{2}(4\ln 2 - 0) - \frac{1}{2}\int_1^2 x\,dx$

$= \frac{1}{2} 4 \ln 2 - \frac{1}{2}\frac{1}{2}x^2 \Big|_1^2$

$= 2\ln 2 - \frac{1}{4}(4 - 1)$

$= 2\ln 2 - \frac{1}{4}(3)$

$= 2\ln 2 - \frac{3}{4}$

#36. $f(x) = \int x\sqrt{x+1}\, dx$ @

$u = x \qquad dv = (x+1)^{\frac{1}{2}} dx$

$du = dx \qquad v = \frac{2}{3}(x+1)^{\frac{3}{2}}$

@ (3,6)

$= \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{2}{3}\int (x+1)^{\frac{3}{2}} dx$

$= \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{2}{3}\frac{2}{5}(x+1)^{\frac{5}{2}} + C$

$= \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} + C$

$f(3) = \frac{2}{3}(3)(4)^{\frac{3}{2}} - \frac{4}{15}(4)^{\frac{5}{2}} + C$

$= 16 - \frac{128}{15} + C$

$= \frac{16 \cdot 15 - 16 \cdot 8}{15} + C$

$= \frac{16 \cdot 7}{15} + C$

$\frac{90}{15} - \frac{112}{15} = C$

$C = \frac{-22}{15}$

128
4 32
16 2

16
7
112

$\therefore f(x) = \frac{2}{3}x(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} - \frac{22}{15}$

#36 another way

$$f(x) = \int x\sqrt{x+1}\,dx \qquad u = x+1 \qquad x = u-1$$
$$du = dx$$

$$= \int \sqrt{u}\,(u-1)\,du$$

$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$$

$$f(3) = \frac{2}{5}(4)^{\frac{5}{2}} - \frac{2}{3}(4)^{\frac{3}{2}} + C$$

$$6 = \frac{64}{5} - \frac{16}{3} + C$$

$$6 - \frac{64}{5} + \frac{16}{3} = C$$

$$\frac{90}{15} + \frac{80}{15} - \frac{192}{15} = C$$

$$\frac{170 - 192}{15} = C$$

$$\frac{-22}{15} = C$$

$$\therefore f(x) = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{22}{15}$$