8.3

#2 $f(x,y) = x^2 - xy + y^2 + 1$

$f_x(x,y) = 2x - y$

$f_y(x,y) = -x + 2y$

$0 = 2x - y$ ①

$0 = -x + 2y$ ②

$x = 2y$

② → ① $0 = 2(2y) - y$

$0 = 4y - y$

$= 3y$

$y = 0$

$(x,y) = (0,0)$

$f(0,0) = 1$

$f_{xx}(x,y) = 2$ $f_{yy}(x,y) = 2$ $f_{xy}(x,y) = -1$

$D(0,0) = f_{xx}(0,0) f_{yy}(0,0) - f_{xy}^2(0,0)$

$= (2)(2) - 1$

$= 3$

$D(0,0) > 0$ & $f_{xx}(0,0) > 0$

rel ~~mo~~ min @ $f(0,0) = 1$

#14. $f(x,y) = \frac{x}{y^2} + xy$

$f_x(x,y) = \frac{1}{y^2} + y \rightarrow 0 = \frac{1}{y^2} + y$ (1)

$f_y(x,y) = -\frac{2x}{y^3} + x \rightarrow 0 = \frac{-2x}{y^3} + x$ (2)

(1) $0 = 1 + y^3$

$y^3 = -1$

$y = -1$

(2) $0 = 2x + x$

$= 3x$

$0 = x$

$(x,y) = (0,-1)$

$f(0,-1) = 0$

$f_{xx}(x,y) = 0$ $\quad f_{yy}(x,y) = \frac{6x}{y^4}$ $\quad f_{xy}(x,y) = -\frac{2}{y^3} + 1$

$f_{xx}(0,-1) = 0$ $\quad f_{yy}(0,-1) = 0$ $\quad f_{xy}(0,-1) = 3$

$D(0,-1) = f_{xx}(0,-1) f_{yy}(0,-1) - f_{xy}^2(0,-1)$

$= 0 - 9$

$= -9$

$D(0,1) < 0$ ∴ saddle pt @ $f(0,-1) = 0$

#20. $f(x,y) = xy + \ln x + 2y^2$

$f_x(x,y) = y + \frac{1}{x} \rightarrow 0 = y + \frac{1}{x}$ ①

$f_y(x,y) = x + 4y \rightarrow 0 = x + 4y$ ②

① $y = -\frac{1}{x}$

① → ② $\quad 0 = x + 4\left(-\frac{1}{x}\right)$

$0 = x^2 - 4$

$x = \pm 2$

$x \neq -2$ because $\ln(-2)$ DNE

$\therefore x = 2$

$(x,y) = (2, -\frac{1}{2})$

$f(2, -\frac{1}{2}) = -1 + \ln 2 + \frac{1}{2}$

$= -\frac{1}{2} + \ln 2$

$f_{xx}(x,y) = -\frac{1}{x^2} \quad f_{yy}(x,y) = 4 \quad f_{xy}(x,y) = 1$

$f_{xx}(2,-\frac{1}{2}) = -\frac{1}{4} \quad f_{yy}(2,-\frac{1}{2}) = 4 \quad f_{xy}(2,-\frac{1}{2}) = 1$

$D(2,-\frac{1}{2}) = f_{xx}(2,-\frac{1}{2})\, f_{yy}(2,-\frac{1}{2}) - f_{xy}^2(2,-\frac{1}{2})$

$= \left(-\frac{1}{4}\right)(4) - 1$

$= -2$

$D(2,-\frac{1}{2}) < -2$ saddle pt @ $f(2, -\frac{1}{2}) = -\frac{1}{2} + \ln 2$

24 Profit = Rev. - Cost.

$P(x) = R(x) - C(x)$

Cost of the german $(2000 - 150p + 100q) \cdot \$4$

Cost of the italin $(1000 + 80p - 120q) \cdot \$3$

$Rev_G \quad p(2000 - 150p + 100q)$

$Rev_I \quad q(1000 + 80p + 120q)$

$$\therefore P(p,q) = 2000p - 150p^2 + 100pq + 1000q + 80pq - 120q^2$$
$$- 4(2000 - 150p + 100q) - 3(1000 + 80p - 120q)$$

$$= 2000p + 600p - 240p + 1000q - 400q + 360q$$
$$+ 100pq + 80pq - 150p^2 - 120q^2 - 8000 - 3000$$

$$= 2360p + 960q + 180pq - 150p^2 - 120q^2 - 11000$$

$P_p = 2360 + 180q - 300p \rightarrow 0 = 2360 + 180q - 300p$

$P_q = 960 + 180p - 240q \rightarrow 0 = 960 + 180p - 240q$

$0 = 236 + 18q - 30p \rightarrow 0 = 118 + 9q - 15p$

$0 = 96 + 18p - 24q \rightarrow 0 = 48 + 9p - 12q$

① $0 = 118 + 9q - 15p$

② $0 = 16 + 3p - 4q$

$3p = 4q - 16$

② → ①

$$0 = 118 + 9q \cancel{+} 15p$$
$$= 118 + 9q - 5(4q - 16)$$
$$= 118 + 9q - 20q + 80$$
$$= 198 - 11q$$
$$q = \frac{198}{11}$$
$$= 18$$

if $q = 18$ $\quad 3p = 4(18) - 16$
$$= 72 - 16$$
$$= 56$$
$$p = \frac{56}{3}$$

$\therefore P\left(\frac{56}{3}, 18\right)$

$$\begin{array}{r} {}^{6} \\ 18 \\ 18 \\ \hline 324 \end{array}$$

$P_{pp} = -300 \qquad P_{qq} = -240 \qquad P_{pq} = 180$

$$D\left(\frac{56}{3}, 18\right) = (-300)(-240) - 180^2$$
$$= 72000 - 32400$$
$$= 39{,}600$$

$D > 0$ & $P_{pp} < 0$

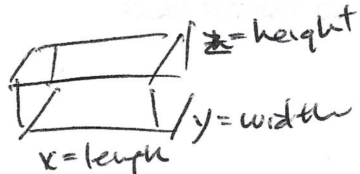
rel max

German \$18.67/bottles

Italian \$18 / bottle

$Vol = xyz$

#32



$$\text{Surface area} = xy + 2xz + 2yz$$

$$300 = xy + 2xz + 2yz$$

$$300 - xy = 2z(x+y)$$

$$z = \frac{300 - xy}{2(x+y)}$$

$$V = xyz$$

$$= \left(\frac{xy}{2}\right)\left(\frac{300 - xy}{x+y}\right)$$

$$= \frac{300xy - x^2y^2}{2(x+y)}$$

$$V_x = \frac{2(300y - 2xy^2)(x+y) - (300xy - x^2y^2)(2)}{2(x+y)^2}$$

$$= \frac{2(\cancel{300xy} + 300y^2 - 2x^2y^2 - 2xy^3 - \cancel{300xy} + x^2y^2)}{2(x+y)^2}$$

$$= \frac{300y^2 - x^2y^2 - 2xy^3}{(x+y)^2} \quad (1)$$

$$V_y = \frac{2((300x - 2x^2y)(x+y) - (300xy - x^2y^2)(1))}{2(x+y)^2}$$

$$= \frac{300x^2 + \cancel{300xy} - 2x^3y - 2x^2y^2 - \cancel{300xy} + x^2y^2}{(x+y)^2}$$

$$= \frac{300x^2 - 2x^3y - x^2y^2}{(x+y)^2} \quad (2)$$

Set $V_x = 0 \quad V_y = 0$

$$① \quad 0 = \frac{300y^2 - x^2y^2 - 2xy^3}{(x+y)^2}$$

$$= \frac{y^2(300 - x^2) - 2xy^3}{(x+y)^2}$$

$$= y^2(300 - x^2) - 2xy^3$$

$$2xy^3 = y^2(300 - x^2)$$

$$2xy = 300 - x^2$$

$$y = \frac{300 - x^2}{2x}$$

$$② \quad 0 = 300x^2 - 2x^3\left(\frac{300 - x^2}{2x}\right) - x^2\left(\frac{300 - x^2}{2x}\right)^2$$

$$= 300x^2 - x^2(300 - x^2) - \frac{1}{4}(300 - x^2)^2$$

$$= 300x^2 - 300x^2 + x^4 - \frac{1}{4}(300 - x^2)^2$$

$$= x^4 - \frac{1}{4}(300 - x^2)^2$$

$$= x^4 - \frac{1}{4}(300^2 - 600x^2 + x^4)$$

$$= x^4 - \frac{1}{4}\cdot 300^2 + 150x^2 - \frac{1}{4}x^4$$

$$= \frac{3}{4}x^4 - \frac{1}{4}\cdot 300^2 + 150x^2$$

$$= 3x^4 - 300^2 + 600x^2$$

$$= x^4 + 200x^2 - 300\cdot 100$$

$$= (x^2 + 300)(x^2 - 100)$$

$x^2 - 100 = 0$

$x = \pm 10$

$x > 0$ so $x = 10$

$x^2 + 300 = 0$
DNE

$$y = \frac{300 - 100}{2(10)} = \frac{200}{2\cdot 10} = 10$$

$$z = \frac{300 - 100}{2(10+10)} = \frac{200}{2\cdot 20} = 5$$

DIM $10 \times 10 \times 5$ $\quad$ $Vol = 500 \text{ in}^3$

$$V_{xx} = \frac{(-2xy^2-2y^3)(x+y)^2-(300y^2-x^2y^2-2xy^3)(2)(x+y)}{(x+y)^4}$$

$$= \frac{2[(-xy^2-y^3)(x+y)-(300y^2-x^2y^2-2xy^3)]}{(x+y)^3}$$

$$V_{xx}(10,10) = \frac{2[(-10\cdot10^2-10^3)(20)-(300\cdot10^2-10^2 10^2-2(10)(10)^3]}{(10+10)^3}$$

$$= \frac{2[10^3(-2)(20)-10^4(3-1-2)]}{(20)^3}$$

$$= \frac{-20\cdot2\cdot10^3\cdot2}{20^3}$$

$$= -10.$$

$$V_{yy} = \frac{(-2x^3-2x^2y)(x+y)^2-(300x^2-2x^3y-x^2y^2)(2)(x+y)}{(x+y)^4}$$

$$V_{yy}(10,10) = \frac{2[(-10^3-10^3)(20)^2-(300\cdot10^2-2\,10^4-10^4)(20)]}{(20)^4}$$

$$= \frac{2(-20^3)(20)^2}{20^4}$$

$$= -10$$

$$V_{xy} = \frac{(600y-2x^2y-6xy^2)(x+y)^2-(300y^2-x^2y^2-2xy^3)(2)(x+y)}{(x+y)^4}$$

$$V_{xy}(10,10) = \frac{(600\cdot10-2\cdot10^3-6\cdot10^3)(20)^2-(300\cdot10^2-10^4-2\cdot10^4)(2)(20)}{(20)^4}$$

$$= \frac{(-2\cdot10^3)(4)(10^2)}{20^4}$$

$$= \frac{10}{2}$$

$$= 5$$

$$D(10,10) = V_{xx}(10,10)\, V_{yy}(10,10) - V_{xy}^2(10,10)$$

$$= (-10)(-10) - 5^2$$

$$= 100 - 25$$

$$= 75$$

$D(10,10) > 0$ & $V_{xx}(10,10) < 0$

rel max @ $V(10,10) = \frac{1}{2} \times 10^3$