8.5

#10. maximize the fn $f(x,y)=\sqrt{y^2-x^2}$

$$x+2y-5=0$$

$$F(x,y,\lambda)=\sqrt{y^2-x^2}+(x+2y-5)\lambda$$

① $F_x=\frac{1}{2}(y^2-x^2)^{-\frac{1}{2}}(-2x)+\lambda$

② $F_y=\frac{1}{2}(y^2-x^2)^{-\frac{1}{2}}(2y)+2\lambda$

③ $F_\lambda = x+2y-5$

Set $F_x, F_y, F_\lambda = 0$

① $0=\frac{-x}{(y^2-x^2)^{\frac{1}{2}}}+\lambda$

$$\lambda=\frac{x}{(y^2-x^2)^{\frac{1}{2}}}$$

① → ② $0=\frac{y}{(y^2-x^2)^{\frac{1}{2}}}+\frac{2x}{(y^2-x^2)^{\frac{1}{2}}}$

$$0=2x+y$$

$$y=-2x$$

③ $0=x+2y-5$

$$0=x+2(-2x)-5$$

$$=-3x-5$$

$$x=-\frac{5}{3}$$

$$y=-2\left(-\frac{5}{3}\right)$$

$$=\frac{10}{3}$$

$\therefore f\left(-\frac{5}{3},\frac{10}{3}\right)=\sqrt{\frac{100}{9}-\frac{25}{9}}$

$$=\frac{\sqrt{75}}{3}$$

$$=\frac{5\sqrt{3}}{3}$$

#12 Find max & min

$$f(x,y) = e^{xy} \qquad x^2+y^2 = 8$$

$$F(x,y,\lambda) = e^{xy} + \lambda(x^2+y^2-8)$$

① $F_x = e^{xy} \cdot y + 2x\lambda$

② $F_y = e^{xy} \cdot x + 2y\lambda$ Set $F_x, F_y, F_\lambda = 0$

③ $F_\lambda = x^2+y^2-8$

① $0 = e^{xy} \cdot y + 2x\lambda$

$2x\lambda = -ye^{xy}$

$2\lambda = -\dfrac{ye^{xy}}{x}$

①→② $0 = e^{xy} \cdot x + y\left(\dfrac{-y\,e^{xy}}{x}\right)$

$0 = e^{xy}x^2 - y^2 e^{xy}$

$= x^2 - y^2$

③ $0 = x^2+y^2-8$

$0 = x^2 - y^2$

$0 = 2x^2 - 8$

$8 = 2x^2$

$4 = x^2$

$x = \pm 2$

if $x=2$ then $y = \pm 2$

if $x=-2$ then $y = \pm 2$

↓ OVER

#12 cont.

$(x,y) = (2,2)$
$= (2,-2)$

$(x,y) = (-2,-2)$
$= (-2,2)$

Max. $f(2,2) = e^4$
$f(-2,-2) = e^4$

Min $f(2,-2) = e^{-4}$
$f(-2,2) = e^{-4}$

#18. $P(x,y) = -\frac{5}{1000}x^2 - \frac{3}{1000}y^2 - \frac{2}{1000}xy + 14x + 12y - 200$

let x = # of deluxe editions

y = # of standard "

Constraint $x + y = 400$

$$F(x,y,\lambda) = -\frac{1}{200}x^2 - \frac{3}{1000}y^2 - \frac{1}{500}xy + 14x + 12y - 200 + \lambda(x+y-400)$$

(1) $F_x = -\frac{1}{100}x - \frac{y}{500} + 14 + \lambda$

(2) $F_y = -\frac{6y}{1000} - \frac{x}{500} + 12 + \lambda$

(3) $F_\lambda = x + y - 400$

set $F_x, F_y, F_\lambda = 0$

(1) $0 = -\frac{1}{100}x - \frac{y}{500} + 14 + \lambda$

(2) $0 = -\frac{3y}{500} - \frac{x}{500} + 12 + \lambda$

(1)-(2)

$0 = \frac{-4x}{500} + \frac{2y}{500} + 2$

$0 = -\frac{x}{125} + \frac{y}{250} + 2 \rightarrow -\frac{x}{125} + \frac{y}{250} = -2$

(3) $x + y = 400 \rightarrow$

$-2x + y = -500$

$2x + 2y = 800$

$3y = 300$

$y = 100$

$x = 300$

300 deluxe edition

100 standard "

$P(300,100) = -\frac{5 \cdot 300^2}{1000} - \frac{3}{1000}(100)^2 - \frac{2}{1000}(100)(300) + 14($

$\downarrow$

18 cont'd

$P(300,100)$

$$= -\frac{1}{200}(300)^2 - \frac{3}{1000}(100)^2 - \frac{1}{500}(300)(100) + 14(300) + 12(100) - 200$$

$$= -\frac{3\cdot 300}{2} - \frac{3\cdot 100}{10} - \frac{300}{5} + 4200 + 1200 - 200$$

$$= -3(150) - 30 - 60 + 4200 + 1000$$

$$= -450 - 90 + 5200$$

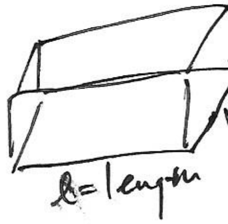
$$= -540 + 5200$$

$$= \$4660$$

300 del.
100 standard
\$4660 max profit.

8.5

#20.



Vol = $l \cdot w \cdot h$

Surface area = bottom + 2 sides + 2 sides

$$V = l \cdot w \cdot h \qquad A = lw + 2lh + 2wh$$

$$48 = lw + 2lh + 2wh$$

$$F(l, w, h, \lambda) = l \cdot w \cdot h + \lambda(lw + 2lh + 2wh - 48)$$

① $F_l = wh + w\lambda + 2h\lambda$

② $F_w = lh + l\lambda + 2h\lambda$

③ $F_h = lw + 2l\lambda + 2w\lambda$

④ $F_\lambda = lw + 2lh + 2wh - 48$

Set $F_l, F_w, F_h, F_\lambda = 0$

① $0 = wh + w\lambda + 2h\lambda$

$-wh = \lambda(w + 2h)$

$\lambda = \dfrac{-wh}{w + 2h}$

② $0 = lh + l\left(\dfrac{-wh}{w+2h}\right) + 2h\left(\dfrac{-wh}{w+2h}\right)$

③ $0 = lw + 2l\left(\dfrac{-wh}{w+2h}\right) + 2w\left(\dfrac{-wh}{w+2h}\right)$

② $0 = lh(w+2h) - lwh - 2wh^2$

③ $0 = lw(w+2h) - 2lwh - 2w^2h$

② $0 = 2lh^2 - 2wh^2$

③ $0 = lw^2 - 2w^2h$ ↓

cont 20.

(2) $0 = lh^2 - wh^2$

$l = w$

if $l = w$

(3) $0 = w^3 - 2w^2 h$

$w^3 = 2w^2 h$

$h = \frac{w}{2} = \frac{l}{2}$

(4) $0 = w^2 + 2w\frac{w}{2} + 2w\left(\frac{w}{2}\right) - 48$

$= w^2 + w^2 + w^2 - 48$

$3w^2 = 48$

$w = \pm 4$

since $w \neq -4$ then $w = 4$

Since $w = 4$ $l = 4$ $h = 2$

$Vol = 4 \times 4 \times 2$

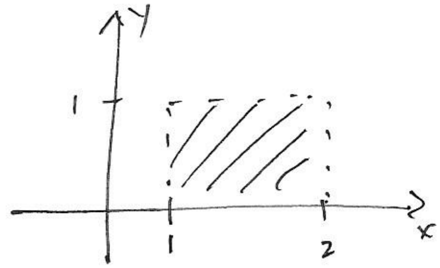
$= 32\ ft^3$ max

8~~7~~

#8 $f(x,y) = \frac{y}{x^2} e^{\frac{y}{x}}$

$1 \le x \le 2$

$0 \le y \le 1$



$\iint_R f(x,y)\,dA$

$= \int_0^1 \int_1^2 \frac{y}{x^2} e^{\frac{y}{x}}\, dx\, dy$

$u = e^{\frac{y}{x}}$

$du = -\frac{y}{x^2} e^{\frac{y}{x}}\, dx$

$= -\int_0^1 \int_{e^y}^{e^{\frac{y}{2}}} du\, dy$

$= -\int_0^1 u \Big|_{e^y}^{e^{\frac{y}{2}}} dy$

$= -\int_0^1 e^{\frac{y}{2}} - e^y\, dy$

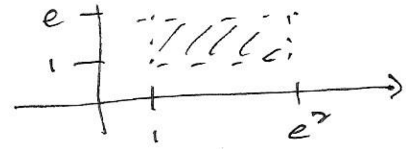
$= -\left(2e^{\frac{y}{2}} - e^y \Big|_0^1 \right)$

$= -\left(2e^{\frac{1}{2}} - 2 - (e-1)\right)$

$= -\left(2e^{\frac{1}{2}} - 2 - e + 1\right)$

$= -\left(2e^{\frac{1}{2}} - e - 1\right)$

#10. $f(x,y) = \dfrac{\ln y}{x}$ $\qquad 1 \le x \le e^2$
$\qquad\qquad\qquad\qquad\quad 1 \le y \le e$



$$\iint_R f(x,y)\,dA$$

$$= \int_1^e \int_1^{e^2} \frac{\ln y}{x}\,dx\,dy$$

$$= \int_1^e \ln y \int_1^{e^2} \frac{1}{x}\,dx\,dy$$

$$= \int_1^e \ln y \ \ln x \Big|_1^{e^2} dy$$

$$= \int_1^e \ln y \ (\ln e^2 - 0)\,dy$$

$u = \ln y \qquad dv = dy$

$du = \frac{1}{y}dy \qquad v = y$

$$= 2\int_1^e \ln y\,dy$$

$$= 2\left(y \ln y \Big|_1^e - \int_1^e dy\right)$$

$$= 2\left(e - 0 - y\Big|_1^e\right)$$

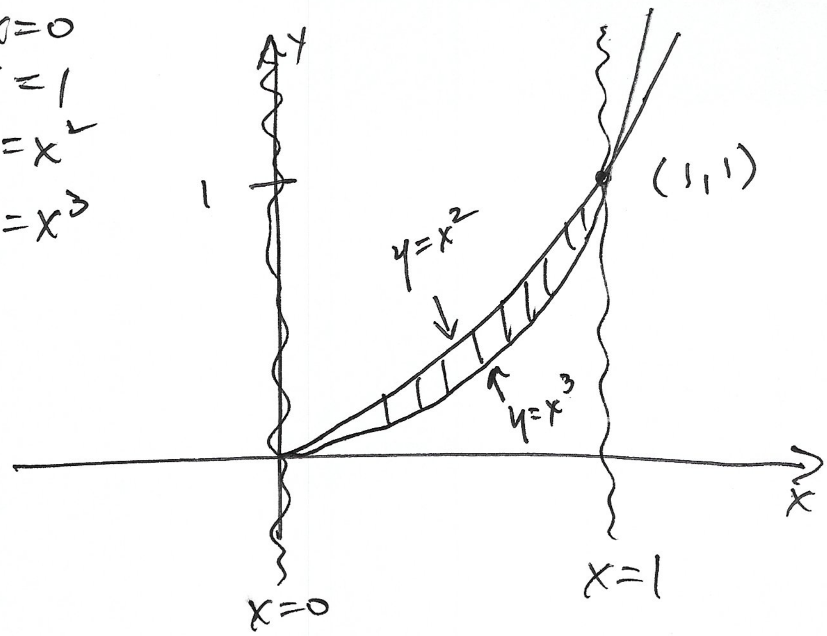
$$= 2(e - (e-1))$$

$$= 2(e - e + 1)$$

$$= 2$$

#16 $f(x,y) = x^2y^2$

$x=0$
$x=1$
$y=x^2$
$y=x^3$



$$\iint_R f(x,y)\,dA$$

$$= \int_0^1 \int_{x^3}^{x^2} x^2 y^2 \, dy \, dx$$

$$= \int_0^1 x^2 \int_{x^3}^{x^2} y^2 \, dy \, dx$$

$$= \int_0^1 x^2 \, \frac{1}{3} y^3 \Big|_{x^3}^{x^2} dx$$

$$= \frac{1}{3}\int_0^1 x^2 (x^6 - x^9)\, dx$$

$$= \frac{1}{3}\int_0^1 x^8 - x^{11} \, dx$$

$$= \frac{1}{3}\left(\frac{1}{9}x^9 - \frac{1}{12}x^{12}\Big|_0^1\right)$$

$$= \frac{1}{3}\left(\frac{1}{9} - \frac{1}{12}\right)$$

$$= \frac{1}{9}\left(\frac{1}{3} - \frac{1}{4}\right)$$

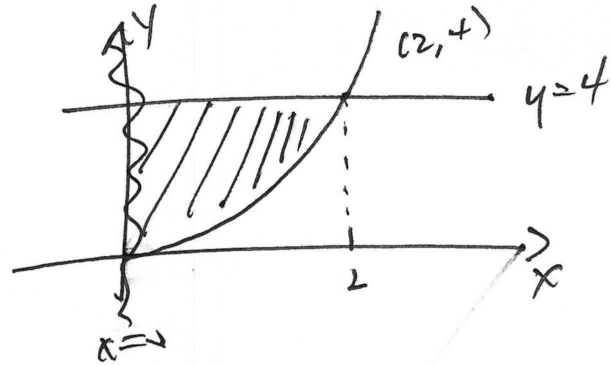
$$= \frac{1}{9}\left(\frac{1}{12}\right)$$

$$= \frac{1}{108}$$

22. 8

$f(x,y) = xe^{-y^2}$

$x=0$
$x=\sqrt{y}$
$y=4$



$\iint_R f(x,y)\, dA$

$= \int_0^4 \int_0^{\sqrt{y}} xe^{-y^2}\, dx\, dy$

$= \int_0^4 e^{-y^2} \frac{1}{2}x^2 \Big|_0^{\sqrt{y}}\, dy$

$= \frac{1}{2} \int_0^4 e^{-y^2} \cdot y\, dy$

$u = -y^2$
$du = -2y\, dy$
$-\frac{1}{2} du = y\, dy$

$= \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \int_0^{-16} e^u\, du$

$= -\frac{1}{4} e^u \Big|_0^{-16}$

$= -\frac{1}{4}\left(e^{-16} - 1\right)$

$= \frac{1}{4}\left(1 - e^{-16}\right)$