9.2 12, 16, 26, 30

#12

$$y' = \frac{xe^x}{2y}$$

$$\frac{dy}{dx} = \frac{xe^x}{2y}$$

$$\int y\,dy = \frac{1}{2}\int xe^x dx$$

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$\frac{1}{2}y^2 = \frac{1}{2}\left(xe^x - \int e^x dx\right)$$

$$y^2 = xe^x - e^x + C$$

16. $$\frac{dy}{dx} = \frac{(x-4)y^4}{x^3(y^2-3)}$$

← IGNORE

$$\int \frac{y^2-3}{y^4}\,dy = \int \frac{x-4}{x^3}\,dx$$

$$\int \frac{1}{y^2}dy - 3\int \frac{1}{y^4}dy = \int \frac{1}{x^2}dx - 4\int \frac{1}{x^3}dx$$

$$-\frac{1}{y} - 3\cdot\frac{1}{-3}\frac{1}{y^3} = -\frac{1}{x} - 4\left(-\frac{1}{2}\right)\frac{1}{x^2} + C$$

$$-\frac{1}{y} + \frac{1}{y^3} = -\frac{1}{x} + \frac{2}{x^2} + C$$

$$\frac{1}{y^3} - \frac{1}{y} = \frac{2}{x^2} - \frac{1}{x} + C$$

9.2 #14. $\frac{dy}{dx} = \frac{xy^2}{\sqrt{1+x^2}}$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{dy}{y^2} = \int \frac{x}{\sqrt{1+x^2}}dx$

$-y^{-1} = \frac{1}{2}\int \frac{1}{\sqrt{u}}du$

$y^{-1} = - u^{\frac{1}{2}} + C$

$y^{-1} = -(1+x^2)^{\frac{1}{2}} + C$

#18. $\frac{dy}{dx} = xe^{-y}$ $y(0)=1$

$\int e^y dy = \int x\,dx$

$e^y = \frac{1}{2}x^2 + C$

$y = \ln(\frac{1}{2}x^2 + C)$

$y(0) = \ln C$

$1 = \ln C$

$e = C$

$\therefore y(x) = \ln(\frac{1}{2}x^2 + e)$

26. $\frac{dy}{dx} = 2xe^{-y}$ $\quad y(0) = 1$

$\int e^y dy = 2\int x\, dx$

$e^y = 2\frac{1}{2}x^2 + C$

$= x^2 + C$

using the initial condition

← IGNORE

$e^1 = C$

$e = C$

$\therefore e^y = x^2 + e$

$y = \ln(x^2 + e)$

30. $\frac{dy}{dx} = 3xy$ $\quad$ passes thru $(0,2)$

$\int \frac{dy}{y} = 3\int x\, dx$

$\ln y = \frac{3}{2}x^2 + c$

$y = e^{\frac{3}{2}x^2 + c}$

$= Ae^{\frac{3}{2}x^2}$ $\qquad e^c = A$

$2 = A$

$\therefore y(x) = 2e^{\frac{3}{2}x^2}$

9.3

#2. let x= bacteria

$$\frac{dx}{dt} = Kx$$

$x(0) = 2000$ ← initial conditions

$x(1) = 5000$

$x(2) = ?$ ← Find

$t = hrs$

$$\int \frac{dx}{x} = k\int dt$$

$$\ln x = Kt + C$$

$$x(t) = e^{Kt+c}$$

$$x(0) = Ae^{K0} \qquad e^{c} = A$$

$$2000 = A$$

$$x(t) = 2000\, e^{Kt} \qquad \text{To find } K$$

$$x(1) = 2000\, e^{K}$$

$$5000 = 2000\, e^{K}$$

$$\frac{5}{2} = e^{K}$$

$$K = \ln\left(\frac{5}{2}\right)$$

$$x(t) = 2000\, e^{\left(\ln\frac{5}{2}\right)t}$$

$$x(2) = 2000\, e^{2\ln\frac{5}{2}}$$

$$= 2000\, e^{\ln\left(\frac{5}{2}\right)^2}$$

$$= 2000\left(\frac{5}{2}\right)^2$$

$$= 25 \cdot 500$$

$= 12500$ bact. after 2hrs

#10. $t = $ yrs
m = mass of radioactive material

$\frac{dm}{dt} = -km$ $\qquad m(0) = m_0 = 100$ mg
$\qquad\qquad\qquad m(5) = 60$ mg
$\qquad\qquad\qquad m(10) = ?$

$\int \frac{dm}{m} = k \int dt$

$\ln m = kt + c$

$m = e^{kt+c}$ $\qquad e^c = A$

$m(0) = A\cancel{e^{-kt}} \to 1$

$100 = A$

$m(t) = 100e^{kt}$ $\qquad$ Find k

$m(5) = 100e^{5k}$

$60 = 100e^{5k}$

$\frac{3}{5} = e^{5k}$

$\ln\left(\frac{3}{5}\right) = 5k$

$k = \frac{1}{5}\ln\left(\frac{3}{5}\right)$ $\qquad$ since the problem is a decay $\ln\left(\frac{3}{5}\right)$ is < 0

$m(t) = 100e^{\frac{1}{5}\ln\left(\frac{3}{5}\right)t}$

$m(10) = 100e^{2\ln\left(\frac{3}{5}\right)}$

$= 100e^{\ln\left(\frac{3}{5}\right)^2}$

$= 100\left(\frac{3}{5}\right)^2$

$= 36$ mg.

#16. $\frac{ds}{dt} = k(D-s)$

$u = D - s$

$du = -ds$

$-du = ds$

$\int \frac{ds}{D-s} = k \int dt$

$-\int \frac{1}{u} du = kt + c$

$-\ln u = kt + c$

$\ln u = -kt - c$

$u = e^{-kt-c}$

$D - s = e^{-kt-c}$

$s(t) = D - e^{-kt-c}$

$e^{-c} = A$

$= D - Ae^{-kt}$

$s(0) = D - A$

$S_0 = D - A$

$D - S_0 = A$

$s(t) = D - (D - S_0)e^{-kt}$

$= S_0 e^{-kt}$

#20. $\frac{dQ}{dt} = kQ(C-Q)$

$C = 400$

$Q(t)$ Find

$Q(0) = 10$

$Q(40) = 45$

$Q(20) = ?$

← IGNORE
MEANT TO GIVE
TO you THIS
PROBLEM
USE IT TO HELP
YOU STUDY

$$\int \frac{dQ}{Q(C-Q)} = \int k\,dt$$

$$\frac{1}{Q(C-Q)} = \frac{A}{Q} + \frac{B}{C-Q}$$

$$1 = A(C-Q) + BQ$$

$$= AC - AQ + BQ$$

$$AC = 1 \qquad -AQ + BQ = 0$$

$$A = \frac{1}{C} \qquad AQ = BQ$$

$$A = B$$

$$\int \frac{dQ}{Q(C-Q)} = \int k\,dt$$

$$\int \frac{1}{CQ}\,dQ + \int \frac{dQ}{C(C-Q)} = k\int dt$$

$$\frac{1}{C}\ln Q + \left(\frac{1}{C}\right)(-)\ln(C-Q) = kt + A$$

$$\frac{1}{C}(\ln Q - \ln(C-Q)) = kt + A$$

$$\ln\left(\frac{Q}{C-Q}\right) = Ckt + A$$

$$\frac{Q}{C-Q} = e^{Ckt+A} \qquad e^{A} = B$$

$$Q = Be^{Ckt}(C-Q)$$

$$= CBe^{Ckt} - BQe^{Ckt}$$

$$Q + BQe^{ckt} = CBe^{ckt}$$

$$Q\left(1 + Be^{ckt}\right) =$$

$$Q(t) = \frac{CBe^{ckt}}{1 + Be^{ckt}} \qquad (Q(0) = 10)$$

$c = 400$

$$Q(0) = \frac{400\,Be^{0}}{1 + Be^{0}}$$

$$10 = \frac{400B}{1+B}$$

$$1 + B = 40B$$

$$1 = 39B$$

$$B = \frac{1}{39}$$

$$Q(t) = \frac{(400)\left(\frac{1}{39}\right)e^{400kt}}{1 + \frac{1}{39}e^{400kt}}$$

$$= \frac{400e^{400kt}}{39 + e^{400kt}}$$

Find K

$$Q(10) = \frac{400e^{400k(10)}}{39 + e^{400k(10)}}$$

$$45\left(39 + e^{4000k}\right) = 400e^{4000k}$$

$$351 + 9e^{4000k} = 80e^{4000k}$$

$$351 = 72e^{4000k}$$

$$e^{4000k} = \frac{351}{72}$$

$$= \frac{117}{24}$$

$$= \frac{39}{8}$$

$$4000k = \ln\left(\frac{39}{8}\right)$$

$$k = \frac{1}{4000} \ln\left(\frac{39}{8}\right)$$

$$Q(t) = \frac{400\, e^{(400)\left(\frac{1}{4000}\right)\ln\left(\frac{39}{8}\right) t}}{39 + e^{(400)\left(\frac{1}{4000}\right)\ln\frac{39}{8} t}}$$

$$= \frac{400\, e^{\left(\frac{1}{10}\right)\left(\ln\frac{39}{8}\right) t}}{39 + e^{\left(\frac{1}{10}\right)\left(\ln\left(\frac{39}{8}\right)\right) t}}$$

$$Q(20) = \frac{400\, e^{2\ln\left(\frac{39}{8}\right)}}{39 + e^{2\ln\frac{39}{8}}}$$

$$= \frac{400\left(\frac{39}{8}\right)^2}{39 + \left(\frac{39}{8}\right)^2}$$

$$= \frac{(400)(39^2)}{39 \cdot 8^2 + 39^2}$$

$$= \frac{400 \cdot 39}{8^2 + 39}$$

$$\begin{array}{r} 64 \\ 39 \\ \hline \end{array}$$

$$= \frac{15600}{103} \text{ fruit flies.}$$

26.

$$\frac{dP}{dt} = cP\ln\left(\frac{L}{P}\right)$$

$$\int \frac{dP}{P\ln\left(\frac{L}{P}\right)} = \int c\,dt$$

$$u = \ln\left(\frac{L}{P}\right)$$

$$du = -\frac{dP}{P}$$

$$-\int \frac{du}{u} = ct + D$$

$$-\ln u = ct + D$$

$$\ln u = -ct - D$$

$$u = e^{-ct-D}$$

$$\ln\left(\frac{L}{P}\right) = Ae^{-ct} \qquad A = e^{-D}$$

$$t=0 \quad \ln\left(\frac{L}{P}\right) = A$$

$$\ln\left(\frac{L}{P_0}\right) = A$$

$$\ln\left(\frac{L}{P}\right) = \ln\left(\frac{L}{P_0}\right)e^{-ct}$$

$$\frac{L}{P} = e^{\ln\left(\frac{L}{P_0}\right)e^{-ct}}$$

$$P(t) = L\,e^{-\ln\left(\frac{L}{P_0}\right)e^{-ct}}$$

$$= L\,e^{\ln\left(\frac{L}{P_0}\right)^{-e^{-ct}}}$$

$$= L\left(\frac{L}{P_0}\right)^{-e^{-ct}}$$

b $\lim_{t\to\infty} P(t)$

$= \lim_{t\to\infty} L\underbrace{\cancel{\left(\frac{L}{P_0}\right)}^{-e^{-\infty}\to 0}}_{\to 1 \text{ as } t\to\infty}$

$= L$

use the original would be best, I think

c) $P'(t) = \frac{dP}{dt}$

$= cP\ln\left(\frac{L}{P}\right)$

$= cP(\ln L - \ln P)$

$P''(t) = cP'(\ln L - \ln P) + cP\left(-\frac{1}{P}\cdot P'\right)$

$= c(cP(\ln L - \ln P)(\ln L - \ln P) - c(cP(\ln L - \ln P))$

$= c^2P\left(\ln\frac{L}{P}\right)\left(\ln\left(\frac{L}{P}\right)\right) - c^2P\left(\ln\frac{L}{P}\right)$

$= c^2P\left(\ln\frac{L}{P}\right)\left(\ln\frac{L}{P} - 1\right)$

Set $P''(t) = 0$

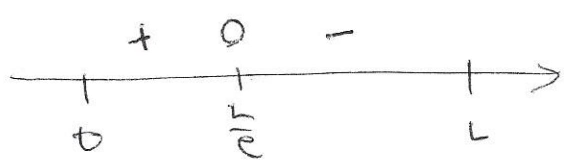
$c^2P = 0$ $\quad$ $\ln\frac{L}{P} = 0$ $\quad$ $\ln\frac{L}{P} - 1 = 0$

$P = 0$ $\quad$ $\frac{L}{P} = 1$ $\quad$ $\ln\frac{L}{P} = 1$

$P = L$

$\frac{L}{P} = e$

$P = \frac{L}{e}$



$\therefore$ $P(t) \nearrow$ rapidly $P = \frac{L}{e}$