

4.4

Properties of log & natural log

let "a" be a pos # w/ $a \neq 1$

let A, B, C be any real # w/ $A > 0$ & $B > 0$

Note: All of these properties also extend to the natural logarithm.

$$1) \log_a(AB) = \log_a A + \log_a B$$

$$\ln(AB) = \ln A + \ln B$$

$$2. \log_a\left(\frac{A}{B}\right) = \log_a A - \log_a B$$

where $B \neq 0$

$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$3. \log_a A^C = C \log_a A$$

$$\ln A^C = C \ln A$$

Note: the logarithm & the natural logarithm is a operator.

$$\text{if } \log_a(3x+1) \neq \log_a 3x + \log_a 1$$

Do Not Do this!

Eval

$$\begin{aligned}\text{Ex \#9 } & \log 50 + \log 250 \\ &= \log (10000) \\ &= \log 10^4 \\ &= 4\end{aligned}$$

we can separate them or
combine them in this case
combine may be easier
w/o the base we know
that this is base 10

$$\begin{aligned}\text{\#15 } & \log_5 \sqrt{5} \\ &= \log_5 5^{\frac{1}{2}} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{\#19 } & \log_4 16^{100} \\ &= \log_4 (4^2)^{100} \\ &= 200\end{aligned}$$

$$\begin{aligned}21. & \log (\log 10^{10000}) \\ &= \log 10000 \\ &= 4\end{aligned}$$

ex. #29 expand

$$\begin{aligned}\log_2 (xy)^{10} \\&= 10 \log_2 (xy) \\&= 10 (\log_2 x + \log_2 y)\end{aligned}$$

ex. #35 $\log_3 \left(\frac{3x^2}{y^3} \right)$

$$\begin{aligned}&= \log_3 (3x^2) - \log_3 y^3 \\&= \log_3 3 + \log_3 x^2 - 3 \log_3 y \\&= 1 + 2 \log_3 x - 3 \log_3 y\end{aligned}$$

#45. $\log \sqrt[4]{x^2 + y^2}$

$$= \frac{1}{4} \log (x^2 + y^2) \leftarrow \text{Don't E}$$

#53 combine

$$\begin{aligned}&4 \log x - \frac{1}{3} \log (x^2 + 1) + 2 \log (x-1) \\&= \log x^4 - \log (x^2 + 1)^{\frac{1}{3}} + \log (x-1)^2 \\&= \log (x^4 (x-1)^2) - \log (x^2 + 1)^{\frac{1}{3}}\end{aligned}$$

move all #s
to exp.

combine + 'd' -

↓

$$= \log \left(\frac{x^4 (x-1)^2}{(x^2+1)^{\frac{1}{3}}} \right)$$

#59 use change of base

$$\begin{aligned} & \log_2 5 \\ &= \frac{\ln 5}{\ln 2} \end{aligned}$$