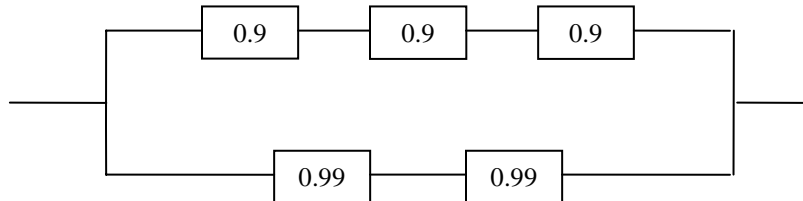


Show your work if you wish to receive credit.

1. The following circuit operates if and only if there is a path of functional devices from left to right. The probability that each device functions is as shown. Assume the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability the device operates? Justify all your calculations.



Solution:

Let  $T_i$  = Event Top Device  $i$  functions,  $i = 1, 2, 3$ . The top path is available if the event  $T = T_1 \cap T_2 \cap T_3$  occurs. Similarly let  $B_i$  = Event Bottom Device  $i$  functions,  $i = 1, 2$ . The top path is available if the event  $B = B_1 \cap B_2$  occurs. By independence

$$P(T) = P(T_1 \cap T_2 \cap T_3) = P(T_1)P(T_2)P(T_3) = (0.9)^3 = 0.729 \text{ and}$$

$$P(B) = P(B_1 \cap B_2) = P(B_1)P(B_2) = (0.99)^2 = 0.9801. \text{ A functional path exists if the event } T \cup B \text{ occurs.}$$

$$P(T \cup B) = P(T) + P(B) - P(T \cap B) = P(T) + P(B) - P(T)P(B) = 0.729 + 0.9801 - 0.7144929 = 0.9946071$$

The step  $P(T \cap B) = P(T)P(B)$  is justified by independence.

2. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2% and 1% of the samples shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment? Show your work and be complete.

Solution: This is problem 2.76, and the solution is posted with other solutions for Section 2.5.