Stat 345-002 Spring 2006 3/2/06 Name (Print!)_________KEY_________

Show your work if you wish to receive credit.

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1. Samples of 20 parts from a metal finishing process are selected every hour. Typically 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.

a) If the percentage of parts that require rework remains at 1%, what is the probability that X exceeds its mean by more than three standard deviations?

$$X \sim Bin(n = 20, p = .01)$$

$$\mu = E(X) = 20(.01) = .2, \quad \sigma = \sqrt{20(.01)(.99)} = .445$$

$$\mu + 3\sigma = .2 + 1.335 = 1.535$$

$$P(X > \mu + 3\sigma) = P(X > 1.535) = P(X > 1) = 1 - P(X \le 1)$$

$$= 1 - [(.99)^{20} + 20(.01)(.99)^{19}] = 1 - [0.8179 + 0.1652]$$

$$= 1 - 0.9831 = 0.0169$$

b) If the rework percentage increases to 4%, what is the probability that X exceeds 1?

$$X \sim Bin(n = 20, p = .04)$$

$$P(X > 1) = 1 - P(X \le 1)$$

$$= 1 - [(.96)^{20} + 20(.04)(.96)^{19}] = 1 - [0.4420 + 0.3683]$$

$$= 1 - 0.8103 = 0.1897$$

2. Suppose the random variable X has a geometric distribution with a mean of 2.5. Calculate P(X > 3).

$$\mu = \frac{1}{p}, \text{ so } p = \frac{1}{\mu}$$

$$\mu = 2.5 \Rightarrow p = \frac{1}{2.5} = 0.4$$

$$P(X > 3) = 1 - P(X \le 3) = 1 - [P(X = 1) + P(X = 2) + P(X = 3)]$$

$$= 1 - [(.6)^0(.4) + (.6)^1(.4) + (.6)^2(.4)] = 1 - (.4)\frac{1 - (.6)^3}{1 - .6} = (.6)^3 = .216$$

3. Return to the setup of problem 1. If the rework percentage is at 1%, how many hours do I expect to collect samples until I finally see one that needs rework?

Each hour I collect 20, so my probability of seeing at least one each hour is

$p = 1 - (.99)^{20} = 1 - .8179 = 0.1821$. If $Y =$ hours to see first, then $Y \sim Geometric(p = 0.1821)$.

$$E(Y) = \frac{1}{.1821} = 5.49 \text{ hours}$$