Stat 345-002 Spring 2006 3/23/06 Name (Print!)____________KEY______________
Quiz 6

Show your work if you wish to receive credit.

1. Suppose that $f(x) = \begin{cases} 1.5x^2, -1 < x < 1 \\ 0, \quad \text{o.w} \end{cases}$.

a) Find P(-0.5 < X < 0.5)

$$P(-0.5 < X < 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 0.5x^3 \Big|_{-0.5}^{0.5} = 2(0.5)^4 = (0.5)^3 = 0.125$$

b) Find x such that P(X > x) = 0.05

$$P(X > x) = \int_x^1 1.5w^2 dw = 0.5w^3 \Big|_x^1 = 0.5(1 - x^3) = 0.05$$

$$(1 - x^3) = 0.1$$

$$x^3 = 0.9$$

$$x = (0.9)^{\frac{1}{3}} = 0.9655$$

c) Find the cumulative distribution function F(x) of X. Be sure to define it over the entire real line.

$$P(X \le x) = \int_{-1}^{x} 1.5w^2 dw = 0.5w^3 \Big|_{-1}^{x} = \begin{cases} 0 & x < -1 \\ 0.5(x^3 + 1), & -1 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

OVER

2. A random variable has cumulative distribution function $F(x)=\begin{cases} 0 & x<-2 \\ 0.25x+0.5 & -2\le x<1 \\ 0.5x+0.25 & 1\le x<1.5 \\ 1 & x\ge 1.5 \end{cases}$.

a) What is the p.d.f.?

Differentiate the c.d.f.: $f(x)=\begin{cases} 0.25\,, & -2<x<1 \\ 0.5\,, & 1<x<1.5 \\ 0\,, & o.w. \end{cases}$

b) What is P(X > 2)?

$$P(X>2)=\int_2^{\infty} f(x)dx=\int_2^{\infty} 0dx=0, \text{ or } P(X>2)=1-P(X\le 2)=1-F(2)=1-1=0$$

c) Find the mean and variance of X.

$$\mu=E(X)=\int_{-\infty}^{\infty} xf(x)dx=\int_{-2}^{1} 0.25xdx+\int_{1}^{1.5} 0.5xdx$$

$$=\frac{.25}{2}x^2\Big|_{-2}^{1}+\frac{.5}{2}x^2\Big|_{1}^{1.5}=\frac{.25}{2}-2(.25)+\frac{.5}{2}(2.25)-\frac{.5}{2}=-0.0625$$

$$E(X^2)=\int_{-\infty}^{\infty} x^2 f(x)dx=\int_{-2}^{1} 0.25x^2dx+\int_{1}^{1.5} 0.5x^2dx$$

$$=\frac{.25}{3}x^3\Big|_{-2}^{1}+\frac{.5}{3}x^3\Big|_{1}^{1.5}=\frac{.25}{3}(1+8)+\frac{.5}{3}(3.375-1)=1.1458333$$

$$\sigma^2=V(X)=E(X^2)-\mu^2=1.1458333-(-.0625)^2=1.142$$