STAT 345 (Spring, 2005): Practice Exam II

1. A machine that fills bottles of Coca Cola is within tolerance 98% of the time (although not necessary to complete this problem, assume that "within tolerance" means a bottle has between 11.9 and 12.1 ounces in it). Assuming that bottles are filled independently of each other, let X denote the number of bottles filled until one is filled within tolerance.

   (a) What is the distribution of X?

   (b) Find $P(X = 1)$, $P(X \leq 2)$, and $P(X > 2)$.

   (c) What is the expected number of bottles filled before one is filled within tolerance?

   (d) What is the expected number of bottles filled before two are filled within tolerance?

2. Let's reconsider the bottling machine. Let X be the number of bottles filled within tolerance in a 6-pack of Coke.

   (a) What is the distribution of X?

   (b) What is the expected number of correctly filled bottles in a 6-pack?

   (c) What is the probability that all 6 bottles are *incorrectly* filled, i.e. not filled within tolerance?

   (d) What is the probability that fewer than 2 bottles are *correctly* filled within tolerance?

3. Pretend there is a gilded urn filled with 15 black balls and 5 white balls. You are to randomly draw 10 balls from the urn without replacement. Let X denote the number of white balls in your draw of 10 from the urn.

   (a) What is the distribution of X?

   (b) What is the expected number of white balls drawn from the urn in a sample of size 10?

   (c) What is $P(X = 0)$? What is $P(X \leq 5)$?

   (d) What is the range of X?

4. The number of cracks formed in freshly poured concrete has a Poisson distribution with a mean of 0.01 crack per square foot of concrete. A 200 square-foot driveway has just been poured. Let X be the number of cracks in the driveway.

   (a) What is the distribution of X?

   (b) What is the probability that there are no cracks in the concrete driveway?

   (c) What is the expected number of cracks in the driveway?

   (d) KonKrete, the company that poured the concrete, will fix any cracks that occur. What is the probability that KonKrete will have to fix the driveway?

5. Suppose the continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

   (a) Find the $P(X > 3)$.

   (b) Find the $P(3 < X < 4)$.

   (c) Determine x such that $P(X < x) = 0.1$.

   (d) Sketch the probability density function of X.

(e) Specify completely the cumulative distribution function of X, $F(x) = P(X \leq x)$.

6. X has a continuous uniform distribution over $(-2, 2)$. That is $X \sim U(-2, 2)$.

   (a) Write down the probability density function, $f(x)$ for X.
   (b) Find the expected value of X and the variance of X.
   (c) Determine x such that $P(-x < X < x) = 0.8$.

7. Suppose the continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{else} \end{cases}$$

   (a) Find the expected value of X and the variance of X.
   (b) Find the $P(X > 0.5)$ and $P(0.25 < X < .75)$.
   (c) Specify completely the cumulative distribution function of X, $F(x) = P(X \leq x)$.

8. The loaves of rye-bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assume that the lengths are normally distributed.

   (a) What percentage of loaves are longer than 31.7 centimeters?
   (b) What percentage of loaves are between 29.3 and 33.5 centimeters?
   (c) What percentage of loaves are shorter than 25.5 centimeters?
   (d) Suppose we need to order bags of the appropriate size for the loaves of bread. Find the length which we would expect 99 percent of the loaves to be under.

9. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes in 4 of the next 6 days assuming that days are independent? (Hint: First find the probability that a person is served in less than 3 minutes on a particular day. Then use this probability and what we know about the probability of r events occurring out of n independent bernoulli trials to finish the calculation).

10. Consider an antiquated gasoline engine with one piston. A "misfire" occurs when the fuel/air mixture doesn't ignite and on the engine we're considering this happens with probability $p = 0.2$. Assume that the operation of the engine is a sequence of independent Bernoulli trials, which we will call *cycles*, in which the engine either fires properly (with probability $1 - p = 0.8$) or misfires (with probability $p = 0.2$). Let X be the number of cycles until a misfire occurs.

    (a) What is the distribution of X?
    (b) What is the expected number of cycles before a misfire occurs?
    (c) What is the probability that a misfire occurs on the third cycle? (i.e. what is $P(X = 3)$?)

11. Now let X be the number of misfires out of $n = 20$; this is the number of cycles in a minute.

    (a) What is the distribution of X?
    (b) What is the expected number of misfires in a minute? (i.e. what is $E(X)$?)
    (c) What is the probability that every cycle is a misfire? (i.e. what is $P(X = 20)$?)
    (d) What is the probability that there are no misfires?
    (e) What is the probability that there are 2 or fewer misfires?