Section 2.7 Homework Solutions

2.95)

Let L = Event User is Legitimate, so L' = Event User is Fraudulant

Let O = Event User calls from 2 or more metrop. areas in a day

Told $P(O \mid L) = 0.01, \quad P(O \mid L') = 0.30, \quad P(L') = 0.0001.$

$$\text{Want } P(L' \mid O) = \frac{P(L' \cap O)}{P(O)} = \frac{P(O \mid L')P(L')}{P(O \mid L')P(L') + P(O \mid L)P(L)}$$

$$= \frac{(.3)(.0001)}{(.3)(.0001) + (.01)(.9999)}$$

$$= \frac{.00003}{.00003 + .009999} = \frac{.00003}{.010029} = .003$$

So only .3% of calls made from 2 or more areas are fraudulant.

2.98)

Let D = event the inspector declares an item defective

Let B = event an item actually is bad

Told $P(D \mid B) = 0.99, \quad P(D \mid B') = 0.005, \quad P(B) = 0.009$

a. Want $P(D) = P[(D \cap B) \cup (D \cap B')] = P(D \cap B) + P(D \cap B')$

$$= P(D \mid B)P(B) + P(D \mid B')P(B')$$

$$= (.99)(.009) + (.005)(.991) = .008910 + .004955 = 0.013865$$

b. Want $P(B' \mid D') = \dfrac{P(B' \cap D')}{P(D')} = \dfrac{P(D' \mid B')P(B')}{P(D')} = \dfrac{(1-.005)(1-.009)}{1-.013865}$

$$= .995(.991) / .986135 =$$

$$= 0.9999 \text{ or } 99.99\%$$

2.99)

Let $S =$ event test signals, O, V, C denote events of 3 solvent types.

$P(S \mid O) = .997, \quad P(S \mid V) = .9995, \quad P(S \mid C) = .897, \quad P(O) = .6, \quad P(V) = .27, \quad P(C) = .13$

$$
\begin{aligned}
a)\, P(S) &= P[(S \cap O) \cup (S \cap V) \cup (S \cap C)] = P(S \cap O) + P(S \cap V) + P(S \cap C) \\
&= P(S \mid O)P(O) + P(S \mid V)P(V) + P(S \mid C)P(C) \\
&= .997(.6) + .9995(.27) + .897(.13) = 0.984675
\end{aligned}
$$

$$
b)\, \mathrm{P(C|S)} = \frac{\mathrm{P(C \cap S)}}{\mathrm{P(S)}} = \frac{P(S \mid C)P(C)}{P(S)} = \frac{.897(.13)}{0.984675} = 0.118424861 \text{ or } 11.84\%
$$

Section 3.2 Homework Solutions

3.14)

x	0	1.5	2	3
f(x)	1/3	1/3	1/6	1/6

a) P(X=1.5) = f(1.5) = 1/3

b) $P(.5 < X < 2.7) = \sum_{.5<x<2.7} f(x) = f(1.5) + f(2) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

c) $P(X > 3) = \sum_{x>3} f(x) = 0$

d) $P(0 \le X < 2) = \sum_{0 \le x < 2} f(x) = f(0) + f(1.5) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

e) $P(X = 0 \text{ or } X = 2) = \sum_{x=0 \text{ or } x=2} f(x) = f(0) + f(2) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

3.16)

x	1	2	3
f(x)	4/7	2/7	1/7

a) $P(X \le 1) = f(1) = 4/7$

b) $P(X > 1) = f(2) + f(3) = 2/7 + 1/7 = 3/7 \quad (= 1 - P(X \le 1))$

c) $P(2 < X \le 6) = f(3) = 1/7$

d) $P(X \le 1 \text{ or } X > 1) = f(1) + f(2) + f(3) = 4/7 + 2/7 + 1/7 = 1$

3.21)

Look at outcomes and calculate probabilities using independence

Outcome	Prob of Outcome	x=#C
NNN	$(.02)^3$	0
CNN	$(.98)(.02)^2$	1
NCN	$(.98)(.02)^2$	1
NNC	$(.98)(.02)^2$	1
CCN	$(.98)^2(.02)$	2
CNC	$(.98)^2(.02)$	2
NCC	$(.98)^2(.02)$	2
CCC	$(.98)^3$	3

This gives the pmf of X:

x	0	1	2	3
f(x)	$(.02)^3$	$3(.98)(.02)^2$	$3(.98)^2(.02)$	$(.98)^3$

Section 3.3 Homework Solutions

3.27

$$F(x)=\begin{cases} 0, & x<-2 \\ 1/8, & -2\le x<-1 \\ 3/8, & -1\le x<0 \\ 5/8, & 0\le x<1 \\ 7/8, & 1\le x<2 \\ 1, & 2\le x \end{cases}$$

a) $P(X \le 1.25) = F(1.25) = 7/8$

b) $P(X \le 2.2) = F(2.2) = 1$

c) $P(-1.1 < X \le 1) = P(X \le 1) - P(X \le -1.1) = F(1) - F(-1.1) = 7/8 - 1/8 = 3/4$

d) $P(X > 0) = 1 - P(X \le 0) = 1 - F(0) = 1 - 5/8 = 3/8$

3.28

$$F(x)=\begin{cases} 0, & x<0 \\ 0.04, & 0\le x<1 \\ 0.16, & 1\le x<2 \\ 0.36, & 2\le x<3 \\ 0.64, & 3\le x<4 \\ 1, & 4\le x \end{cases}$$

a) $P(X < 1.5) = F(1.5^{-}) = .16$ (value just to the left of 1.5)

b) $P(X \le 3) = F(3) = .64$

c) $P(X > 2) = 1 - P(X \le 2) = 1 - F(2) = 1 - .36 = .64$

d) $P(1 < X \le 2) = P(X \le 2) - P(X \le 1) = F(2) - F(1) = .36 - .16 = .20$

3.33

a) $P(X \le 3) = F(3) = 1$

b) $P(X \le 2) = F(2) = .5$

c) $P(1 \le X \le 2) = P(X \le 2) - P(X < 1) = F(2) - F(1^{-}) = .5 - 0 = .5$

d) $P(X > 2) = 1 - P(X \le 2) = 1 - F(2) = 1 - .5 = .5$

Section 3.4 Homework Solutions

3.38

x	0	1.5	2	3
f(x)	1/3	1/3	1/6	1/6

$$E(X) = \mu = \sum_x xf(x) = 0(1/3) + 1.5(1/3) + 2(1/6) + 3(1/6)$$

$$= 0 + 1/2 + 1/3 + 1/2 = 4/3$$

$$E(X^2) = \sum_x x^2 f(x) = 0^2(1/3) + (1.5)^2(1/3) + 2^2(1/6) + 3^2(1/6)$$

$$= 0 + 3/4 + 2/3 + 3/2 = 35/12$$

$$V(X) = \sigma^2 = E(X^2) - \mu^2 = 35/12 - 16/9 = (105 - 64)/36 = 41/36$$

3.39

x	-2	-1	0	1	2
f(x)	1/8	2/8	2/8	2/8	1/8

$$E(X) = \mu = \sum_x xf(x) = (-2)(1/8) + (-1)(2/8) + 0(2/8) + 1(2/8) + 2(1/8)$$

$$= -2/8 - 2/8 + 0 + 2/8 + 2/8 = 0$$

$$E(X^2) = (-2)^2(1/8) + (-1)^2(2/8) + 0^2(2/8) + 1^2(2/8) + 2^2(1/8)$$

$$= 4/8 + 2/8 + 0 + 2/8 + 4/8 = 12/8 = 3/2$$

$$V(X) = \sigma^2 = E(X^2) - \mu^2 = 3/2 - 0 = 3/2$$

3.45

$$6 = E(X) = \mu = \sum_x xf(x) = 0(1/5) + 1(1/5) + 2(1/5) + 3(1/5) + x(1/5)$$

$$= (1/5)(6 + x)$$

So,

$$6 + x = 30$$

$$x = 24$$