Dynamic Fracture Simulations in MPM using a J-Integral Approach

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Outline

- Fracture algorithm
- Experimental Results
- Simulation Results
- Conclusions

Acknowledgements
John Nairn (NairnFEAMPM)
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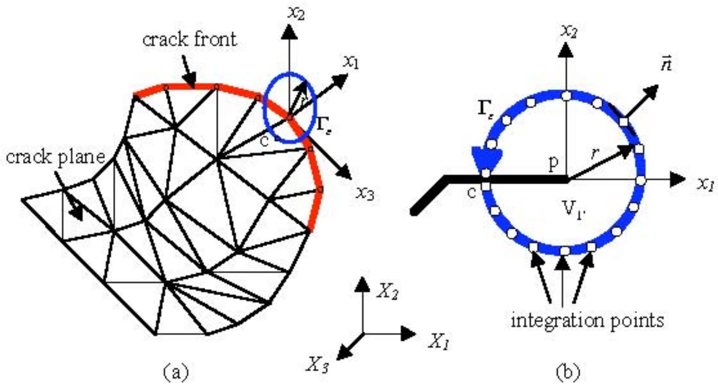
Dynamic fracture approach (Guo and Nairn, 2006)

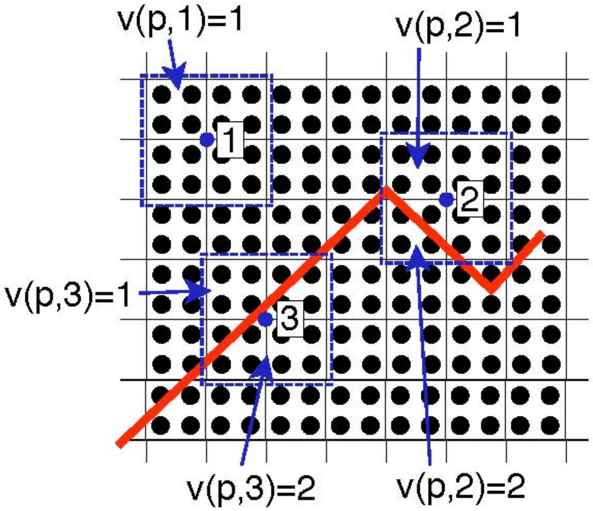


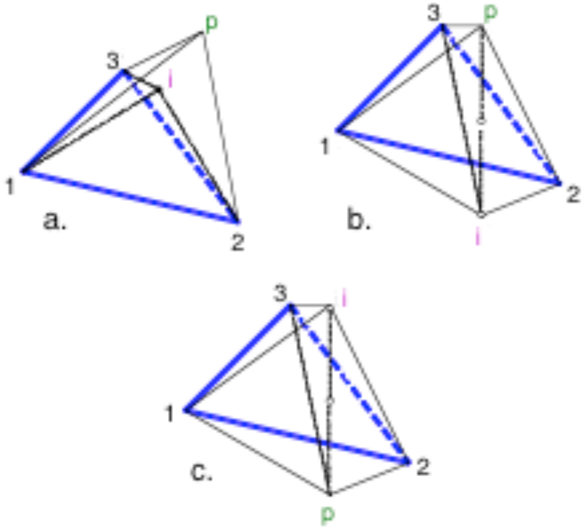
- 3D
- Two velocity fields
- Fracture path arbitrary
  - Requires velocity field/particle correspondence to be determined and updated.
- Dynamic fracture approach
  - Calculate J integrals at crack front.
  - Track triangulated *fracture surface midpoint* using massless particles at triangle vertices.

Dynamic fracture approach (Guo and Nairn, 2006)

Standard GIMP Algorithm modifications

- Determine velocity field of each particle/grid node pair:
  - Particle/node pairs far from the crack are assigned velocity field 1.
  - Particle/node pairs near the crack and *above* are assigned velocity field 1.
  - Particle/node pairs near the crack and *below* are assigned velocity field 2.





Dynamic fracture approach (Guo and Nairn, 2006)

Standard GIMP Algorithm modifications

- Interpolate particle data to the appropriate grid velocity fields:
  - Crack surface contact is handled via modified two velocity field algorithms for stick and Coulomb frictional sliding contact *at grid nodes near the fracture surface*.
  - Crack surface provides exactly opposite normals for the velocity fields (exact conservation of momentum).
  - Particle displacements (in addition to velocities) are used to detect surface overlap.

Dynamic fracture approach (Guo and Nairn, 2006)

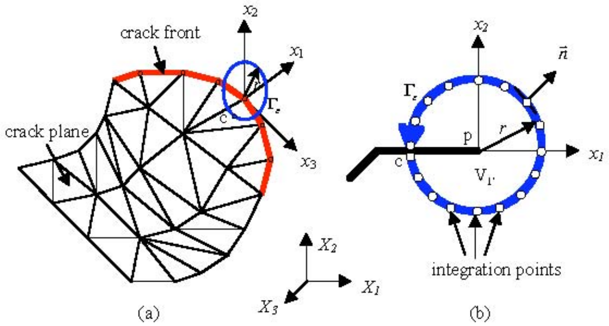
Standard GIMP Algorithm modifications

- Move the crack surface:
  - Crack particles are moved with the updated "center of mass velocity" extrapolated from the grid.

$$v_i^{cm} = \frac{m_i^1 v_i^1 + m_i^2 v_i^2}{m_i^1 + m_i^2}$$

$$v_c = \sum_i v_i^{cm} S_i(x_c)$$

Dynamic fracture approach (Guo and Nairn, 2006)



Standard GIMP Algorithm modification

- Calculate J integrals:

$$J_k = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left[\left(\int \sigma_{ij} d\varepsilon_{ij} + \frac{1}{2} \rho \dot{u}_i \dot{u}_i \right) n_k - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_k} \right] d\Gamma$$

  - Determine crack front coordinate system, and contour Γ_ε (a circle with radius r = twice the grid spacing).
  - Select integration points on the contour (16) in the crack front coordinate system and transform these points to the global coordinate system.
  - Interpolate needed solution terms from grid data in global coordinate system and transform to crack front coordinate system.
  - Numerically integrate J integrals using the midpoint rule.

Dynamic fracture approach (Guo and Nairn, 2006)

Advantages:

- 3-D, large deformations.
- Incorporates traditional fracture mechanics (as opposed to cohesive elements or damage constitutive models).
- Incorporates established approach to crack surface contact.
- Excellent results for crack stress intensity factors relative to FEM solutions and various analytic solutions.
- No introduction of additional length scale/resolution requirements or reliance on AMR for accuracy.

Dynamic fracture approach (Guo and Nairn, 2006)

Advantages:

- Fracture propagation independent of grid and particles.
- Various criteria available for fracture propagation threshold and direction.
  - $G = J_1\cos\theta + J_2\sin\theta > G_{crit}$
- May be combined with cohesive zones for very general modeling approach ("process zone" behind fracture tip).

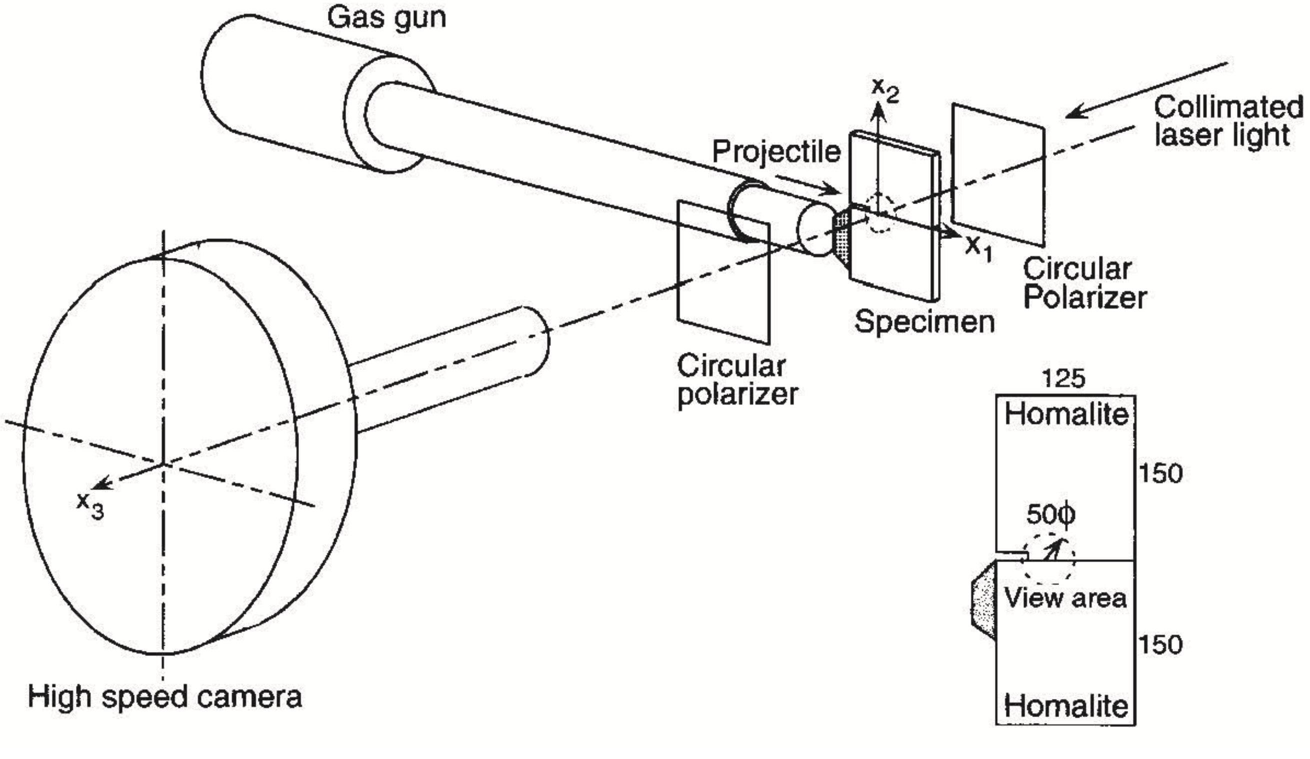
Dynamic fracture approach (Guo and Nairn, 2006)

Disadvantages (relative to cohesive element approach):

- Algorithm complexity.
- Substantial increase in computational effort? (mitigated by reduced need for AMR?).
- Difficulties in handling intersection of crack surface and free surface (no material in which to evaluate the J-integral).

Dynamic Shear Fracture Experiments

Experimental Investigation of fracture propagation along a weak interface (Rosakis et al., 1999, Rosakis, 2002)

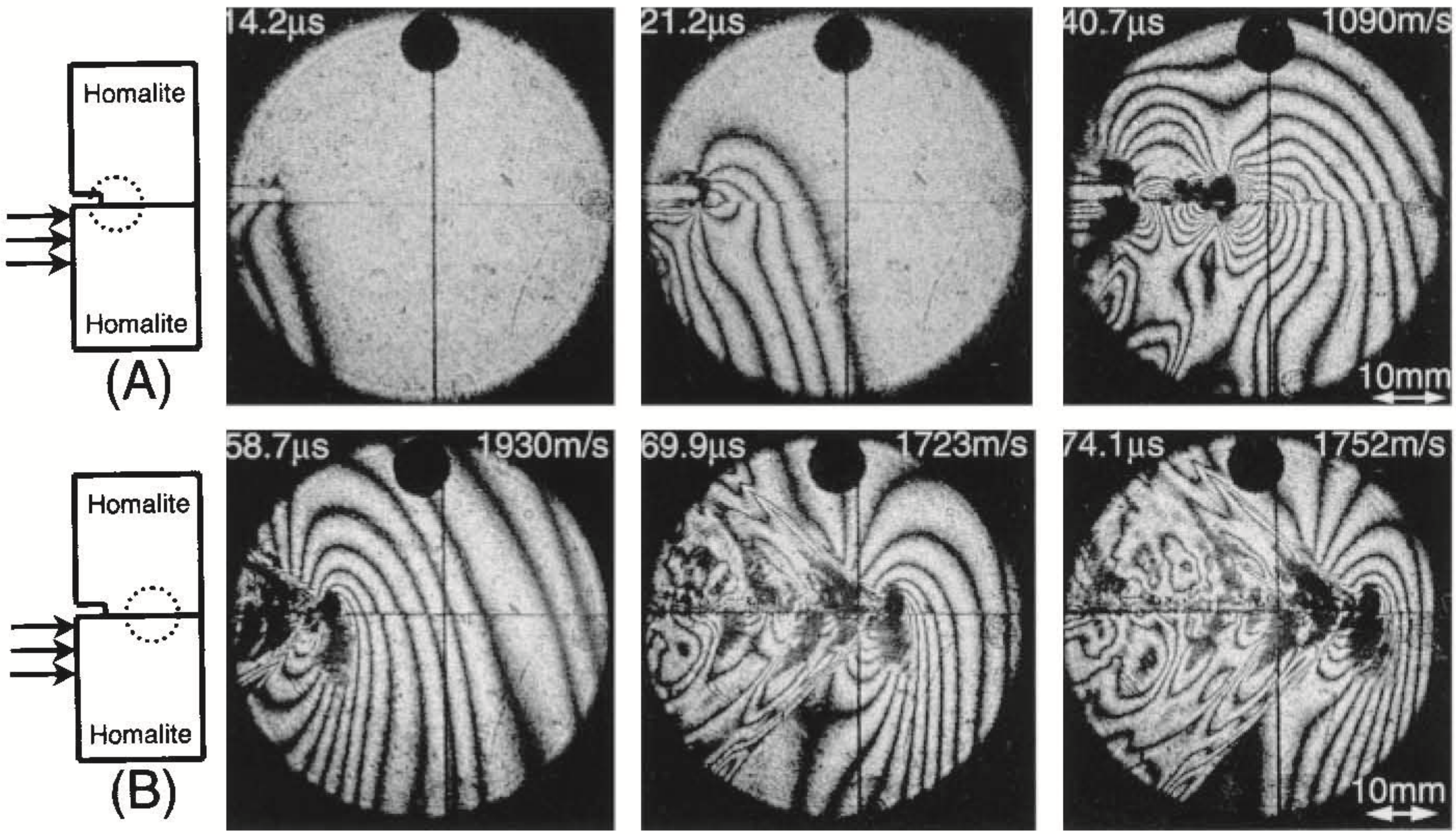


Dynamic Shear Fracture Experiments (Rosakis et al., 1999)

Impact velocity 25 m/s.

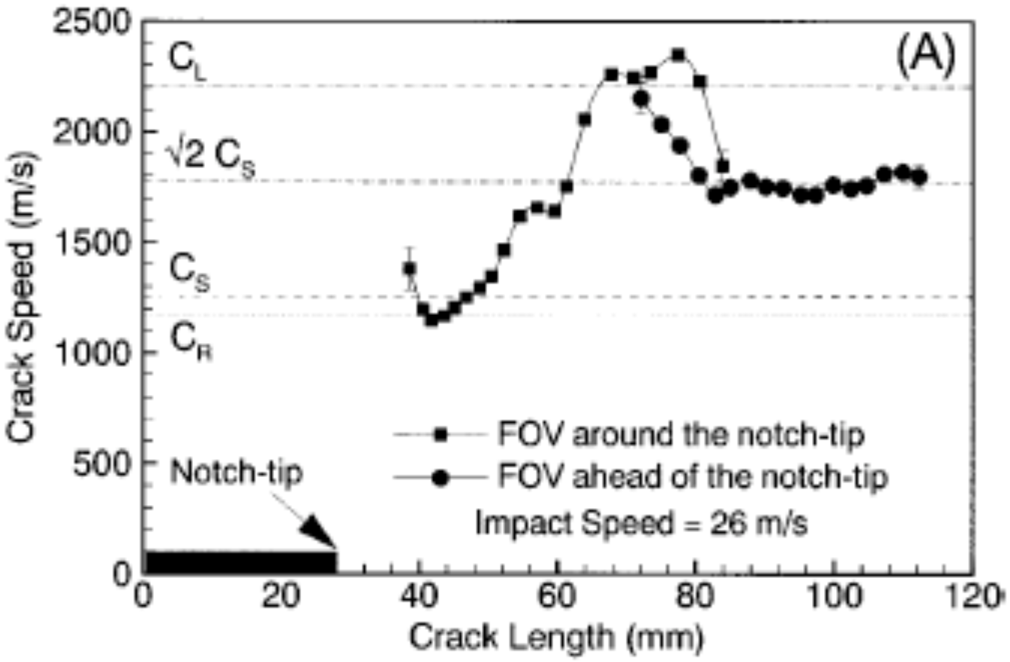
Time after impact

Fracture tip speed



Dynamic Shear Fracture Experiments

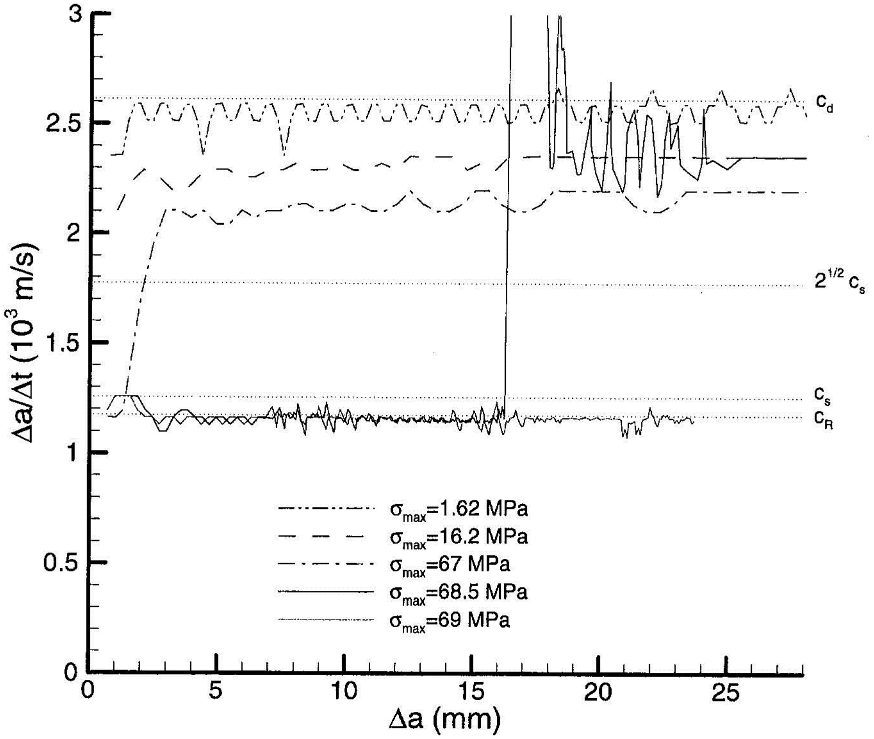
- C_L = longitudinal wave speed
- C_S = shear wave speed
- C_R = Raleigh wave speed
- C_R < crack speed < C_S forbidden
- C_S < crack speed < $\sqrt{2}C_S$ unstable



Rosakis et al. 1999

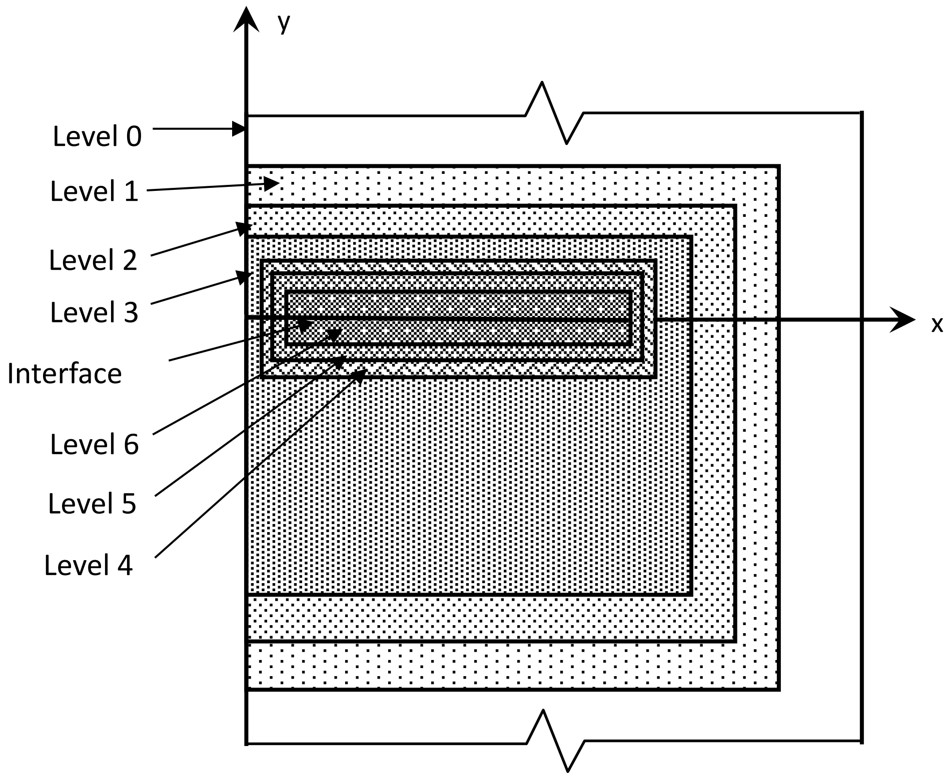
Cohesive Element Simulations

- FEM calculations (Needleman, 1999) using AMR, frictionless contact.
- 2-D plane strain
- Impact velocity 26 m/s, varied cohesion energy
- Velocity jump case corresponds to creation of daughter crack.



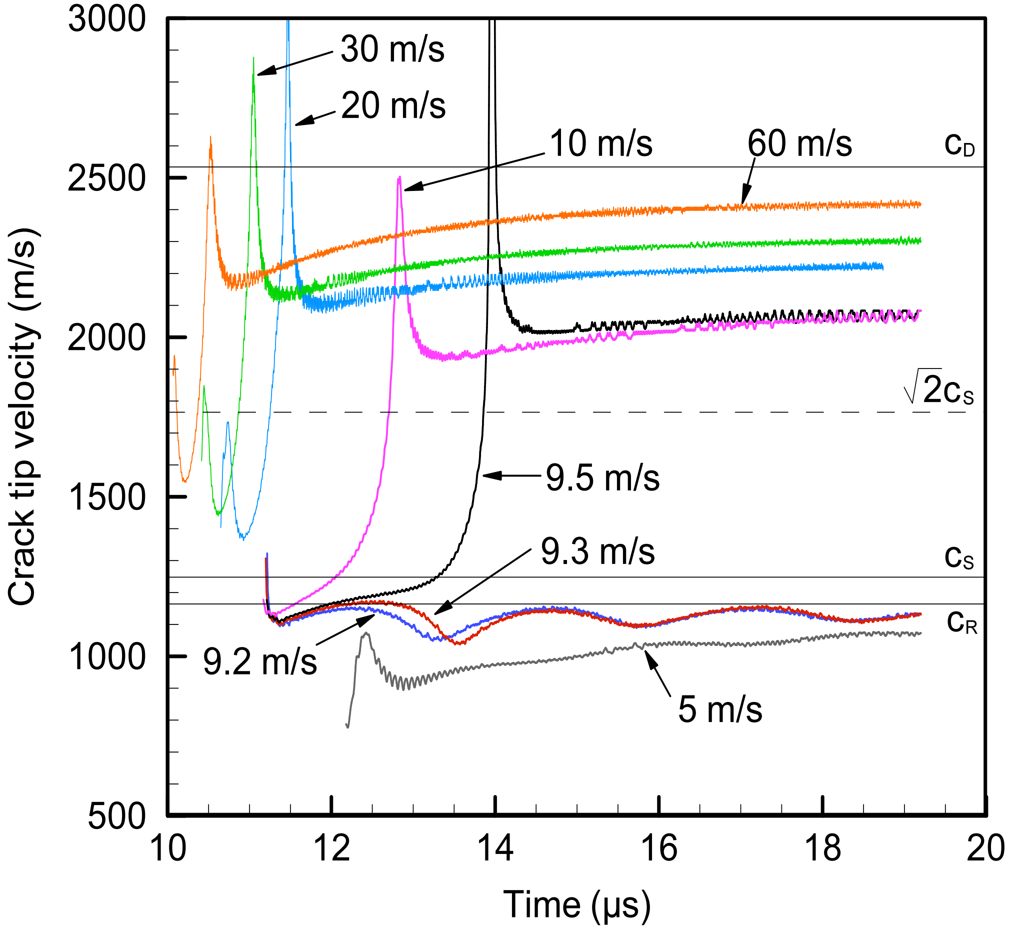
Cohesive Element Simulations

- MPM calculations (Daphalapurkar, 2007) using AMR.
- Decohesion particles track crack surfaces
- No contact conditions post-decohesion



Cohesive Element Simulations

- 2-D plane strain.
- Varied Impact velocity, shear cohesion energy = 26 J/m^2.
- Velocity jump case(9.5 m/s) corresponds to creation of daughter crack.



J-Integral Fracture Simulations

- 2-D plane strain
- J-integral calculated along mesh lines (4x4 cells surrounding crack tip).
- For dynamic fracture, volume integral formulation needed for path independence.

$$J_k = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon} \left[\left(\int \sigma_{ij} d\varepsilon_{ij} + \frac{1}{2} \rho \dot{u}_i \dot{u}_i \right) n_k - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_k} \right] d\Gamma$$

$$= \int_{\Gamma} \left[\left(\int \sigma_{ij} d\varepsilon_{ij} + \frac{1}{2} \rho \dot{u}_i \dot{u}_i \right) n_k - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_k} \right] d\Gamma$$

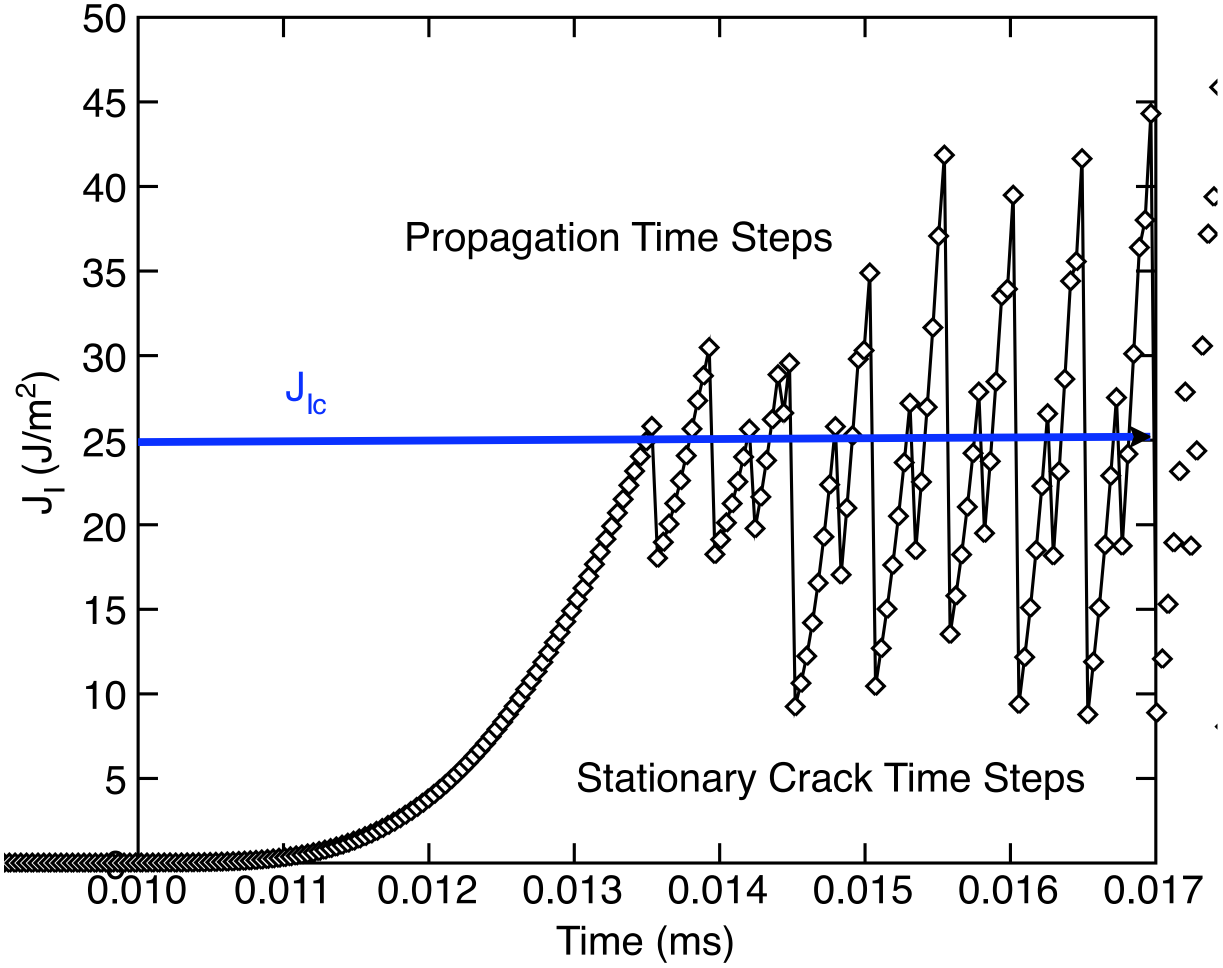
$$+ \int_{A(\Gamma)} \rho \left[\frac{\partial^2 u_i}{\partial t^2} \frac{\partial u_i}{\partial x_m} - \frac{\partial u_i}{\partial t} \frac{\partial^2 u_i}{\partial t \partial x_m} \right] dA$$

J-Integral Fracture Simulations

- Direction of fracture propagation constrained to horizontal (along glue bond).
- Fracture propagation occurs when $J_I > J_{Ic}$.
- Fracture propagates at longitudinal wave speed (during propagation time step).
- Typically several propagation time steps occur sequentially (crossing ~ 1 cell), after which crack tip is stationary for several time steps.

J-Integral Fracture Simulations

Typical Algorithm Performance

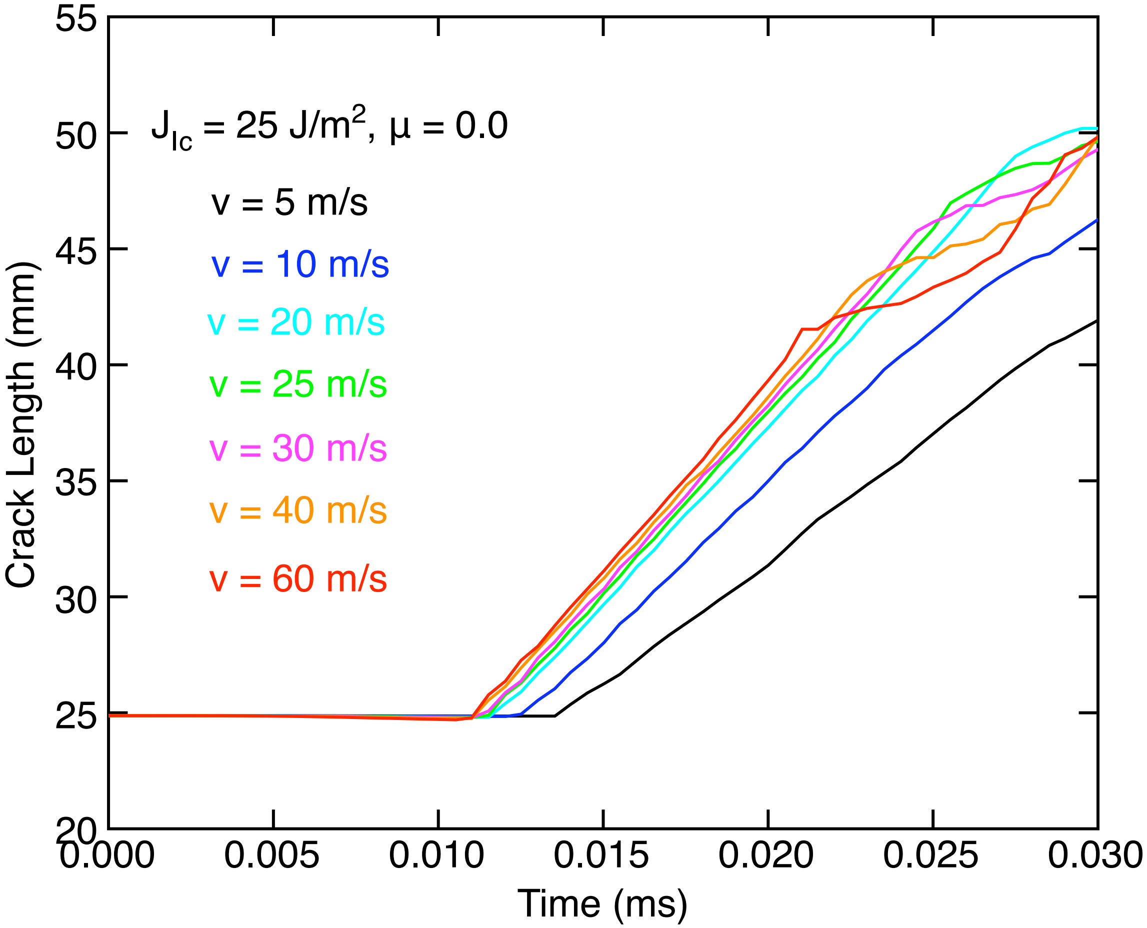


J-Integral Fracture Simulations

- Cell size 0.5mm (Daphalapurkar level 2 resolution; level 0 coarsest, level 6 finest)
- Parameters varied:
  - Critical energy release rate, J_{Ic}.
  - Impact velocity, v.
  - Coefficient of Coulomb friction along fracture surfaces, μ.

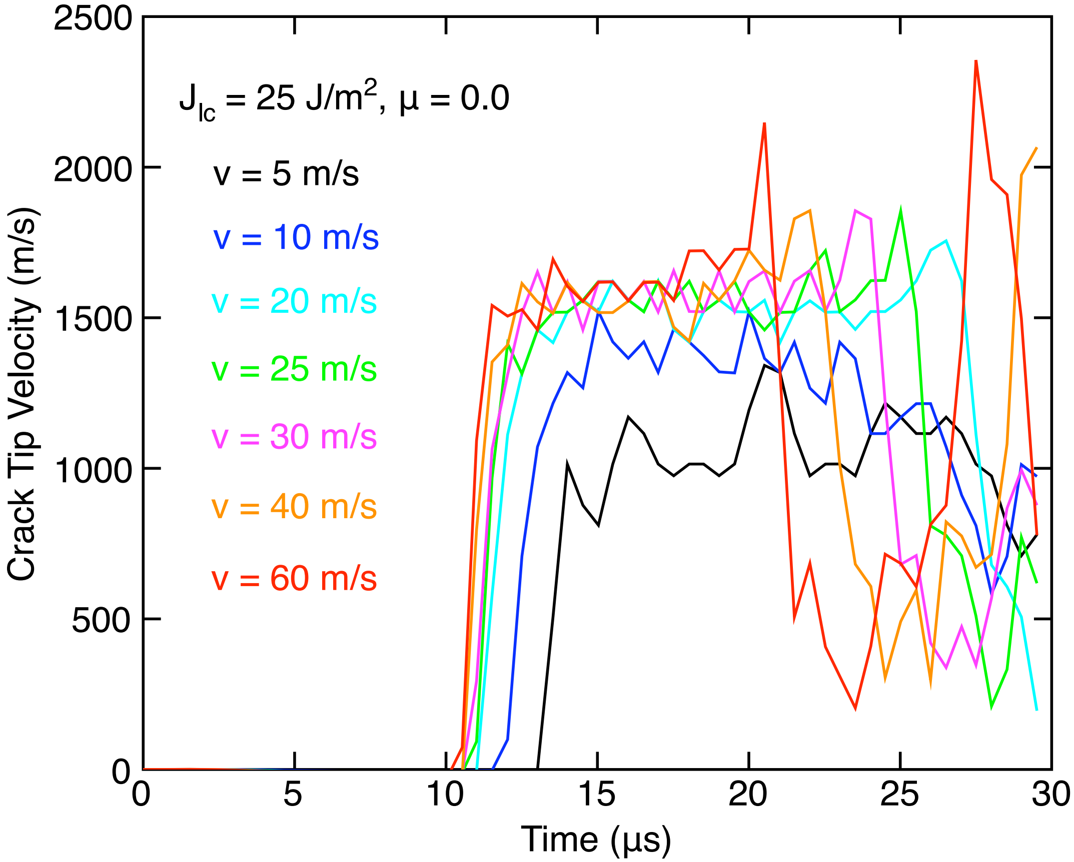
J-Integral Fracture Simulations

- J_{Ic} = 25 J/m^2
- Frictionless crack surfaces
- Steadily increasing propagation velocity (with impact velocity, v) saturates near 1600 m/s.
- “Break-point” crack slow down/arrest near t = 20 μs also seen in Needleman (1999).



J-Integral Fracture Simulations

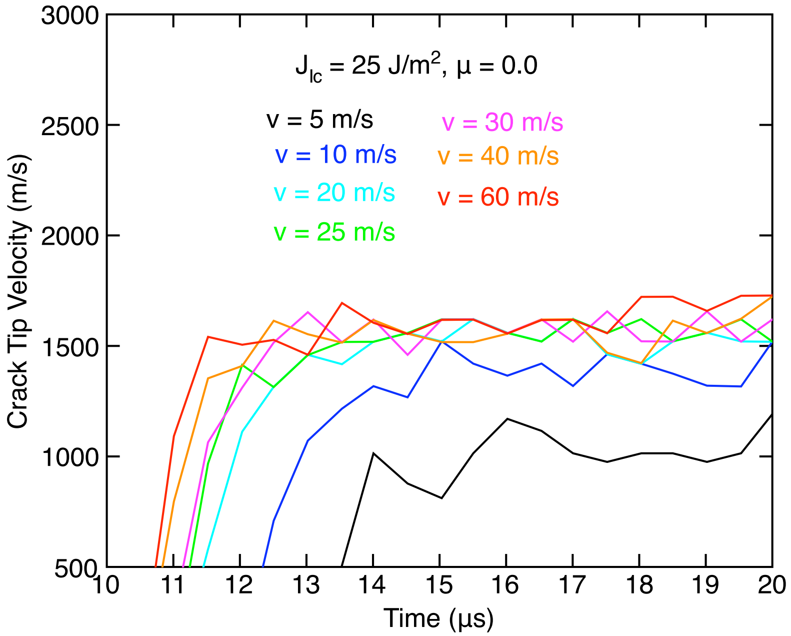
- Recall that during a fracture propagation time step the crack tip speed = C_L.
- However, the crack generally propagates for several time steps and is then stationary for several time steps.
- Crack tip velocity presented here is a *moving box average* over 1 μs.

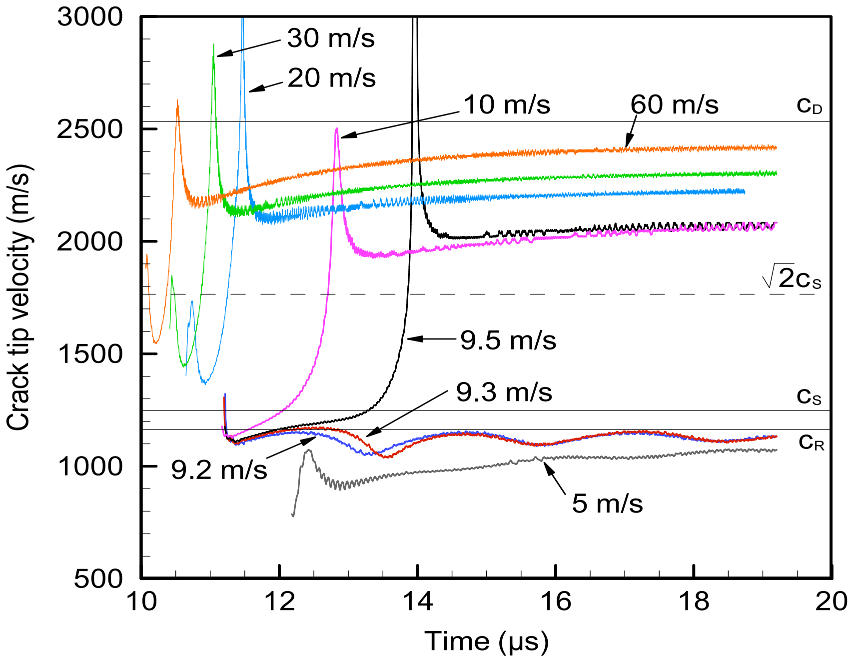


J-Integral Fracture Simulations

Comparison with MPM cohesive zone calculations (Daphalapurkar, 2007)

- J_{Ic} = 25 J/m^2 very similar to decohesion energy in Daphalapurkar (26 J/m^2)
- Similar crack tip velocities at lowest impact speed
- J-integral crack tip speeds slower for higher impact velocities

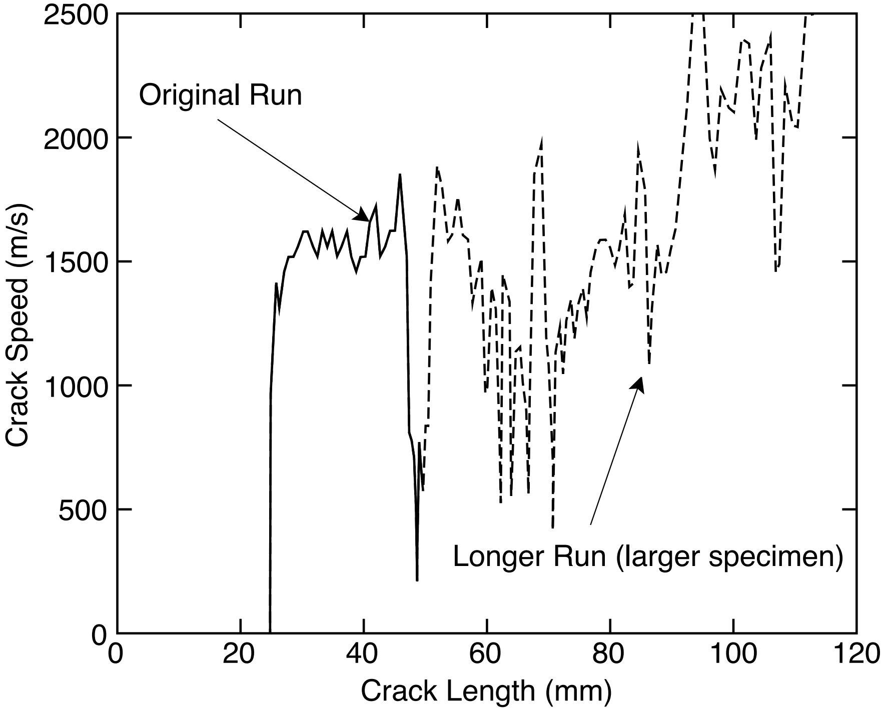


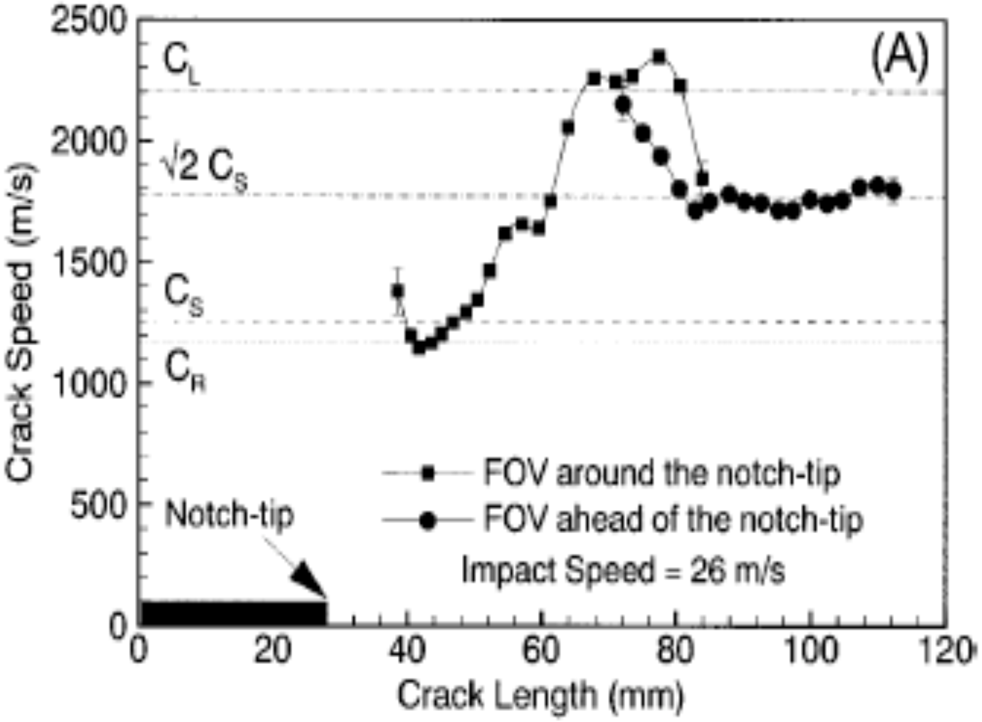


J-Integral Fracture Simulations

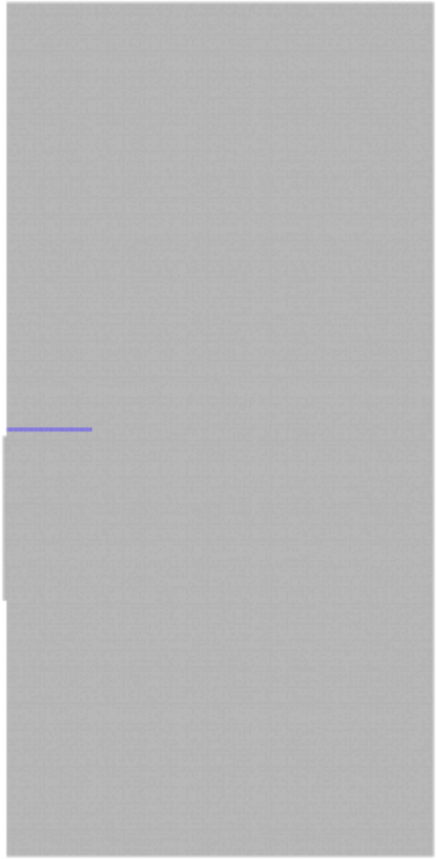
Comparison to experimental results (Rosakis)

- J_{Ic} = 25 J/m^2 (simulations), impact velocity 25 m/s
- Original run used Homalite block ~¼ the size of that used in experiments
- Longer run used 125 mm x 250 mm block (nearly the experimental specimen size)
- Not directly comparable (blunt notch vs sharp crack, plane stress vs. plane strain).



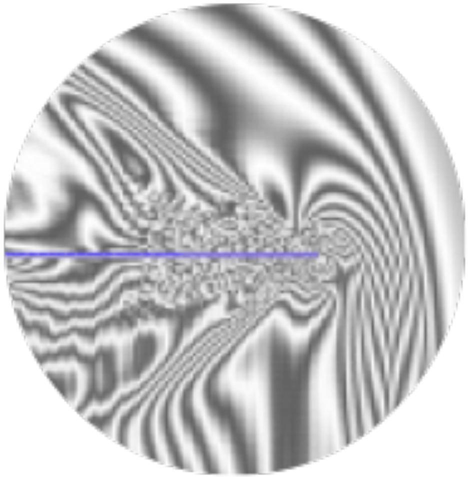


J-Integral Fracture Simulations

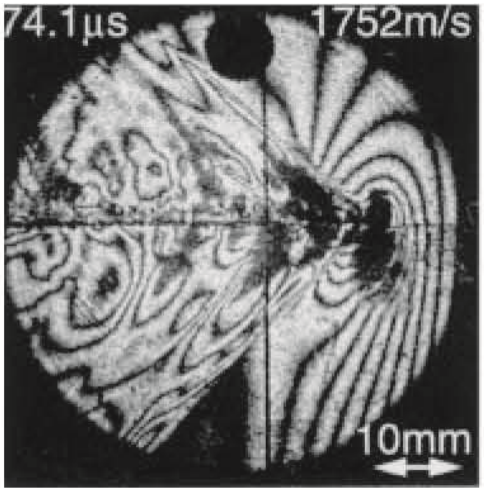


Comparison to experimental results (Rosakis)

- Fringe patterns similar.



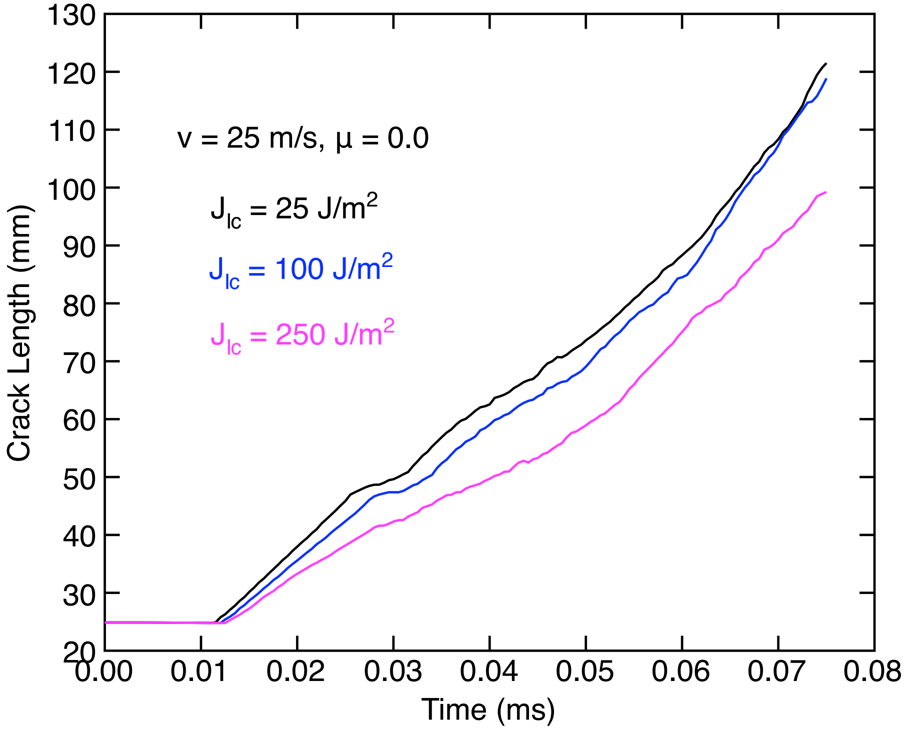
Simulation



Experiment

J-Integral Fracture Simulations

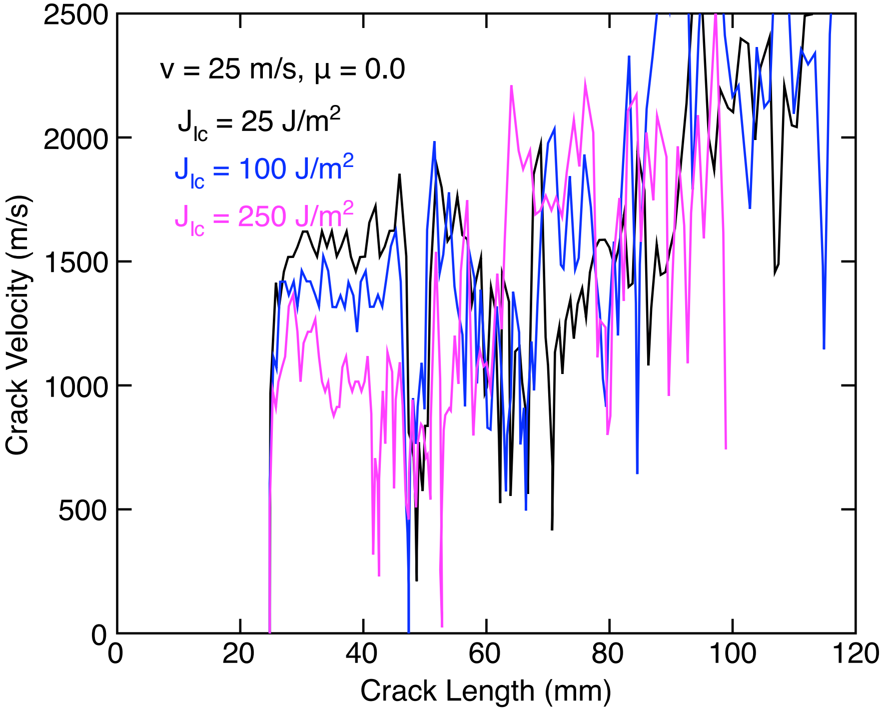
- Increasing J_{Ic} decreases crack tip velocity (as also seen in Needleman's cohesive zone simulations)
- Web search suggests for Homalite $100 < J_{Ic} < 1000$).

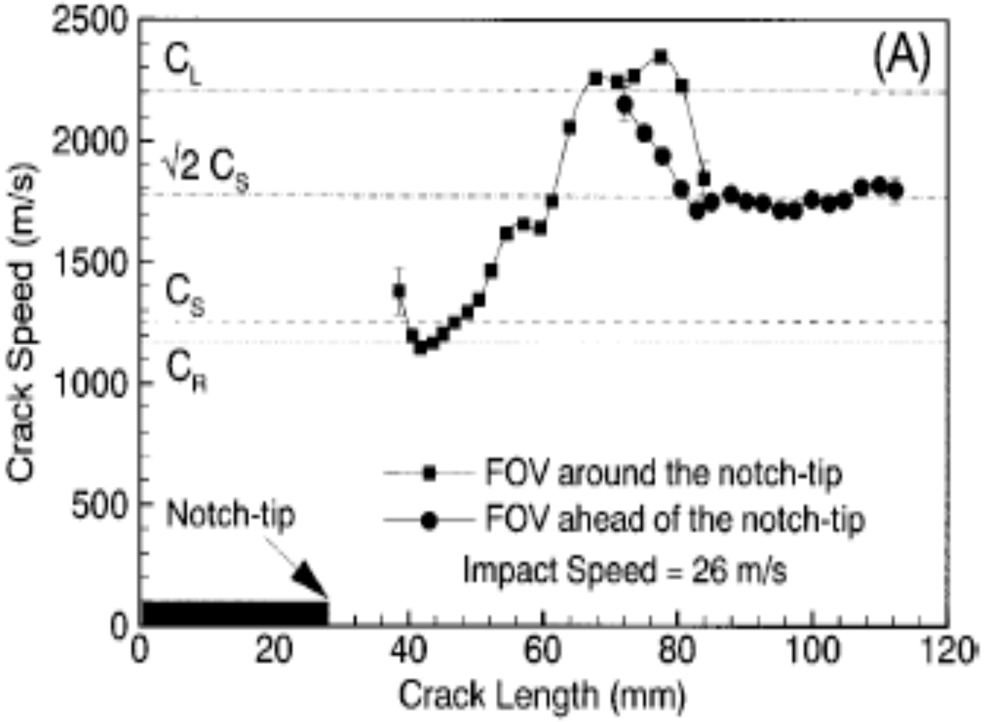


J-Integral Fracture Simulations

Comparison to experimental results (Rosakis)

- Impact velocity 25 m/s, frictionless contact (simulations)
- Longer runs used 125 mm x 250 mm block (nearly the experimental specimen size)
- Vary J_{Ic} (Web search suggests for Homalite $100 < J_{Ic} < 1000$).

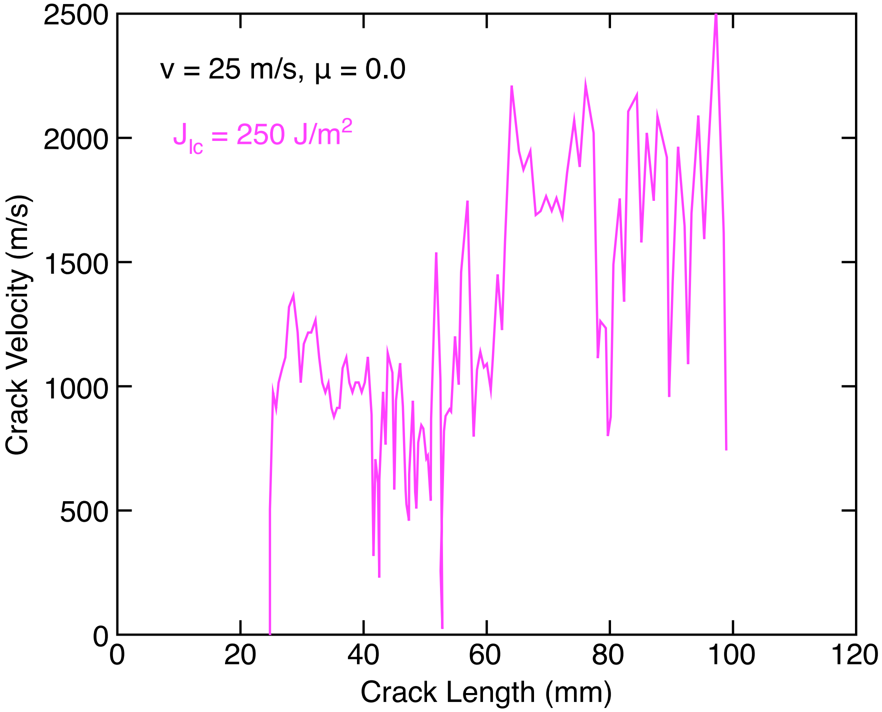


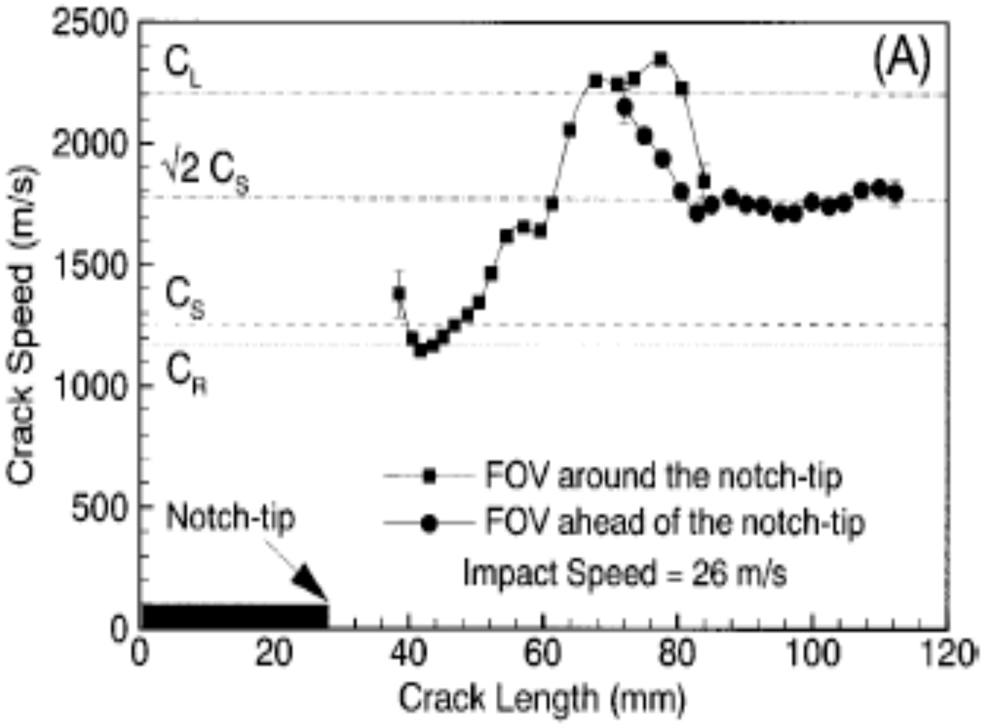


J-Integral Fracture Simulations

Close look at most interesting case

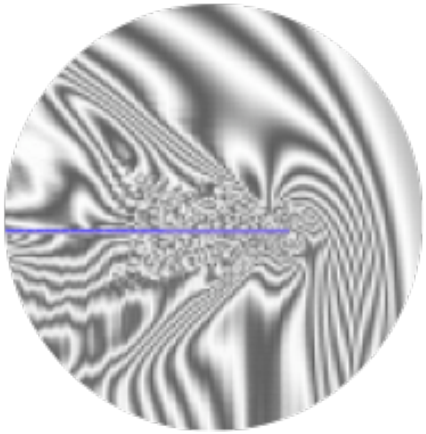
- Impact velocity 25 m/s, J_{Ic} = 250 J/m^2, frictionless contact
- Suggestive of transition case
  - Initially crack tip velocities ~ 1000 m/s
  - Later crack tip velocities ~1800 m/s



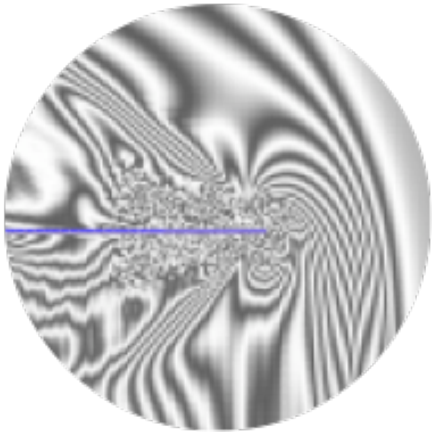


J-Integral Fracture Simulations

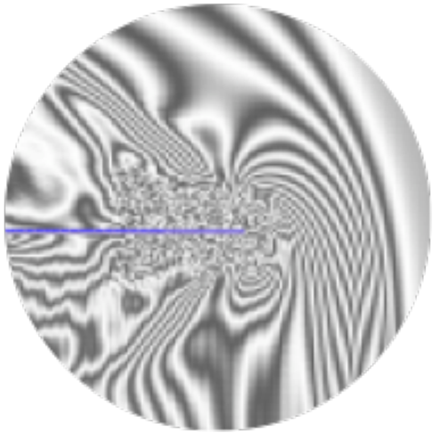
- Increasing J_{Ic} decreases crack tip velocity (as also seen in Needleman's cohesive zone simulations)
- Fringe patterns more similar to experiments for higher J_{Ic}.



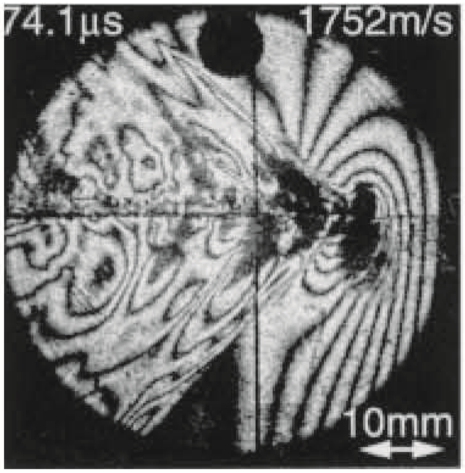
J_{Ic} = 25 J/m^2



J_{Ic} = 100 J/m^2



J_{Ic} = 250 J/m^2

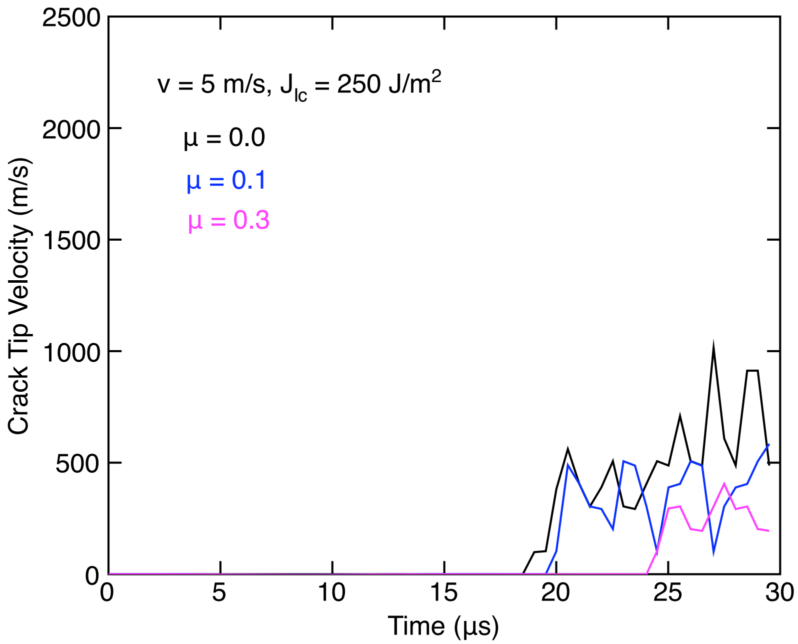


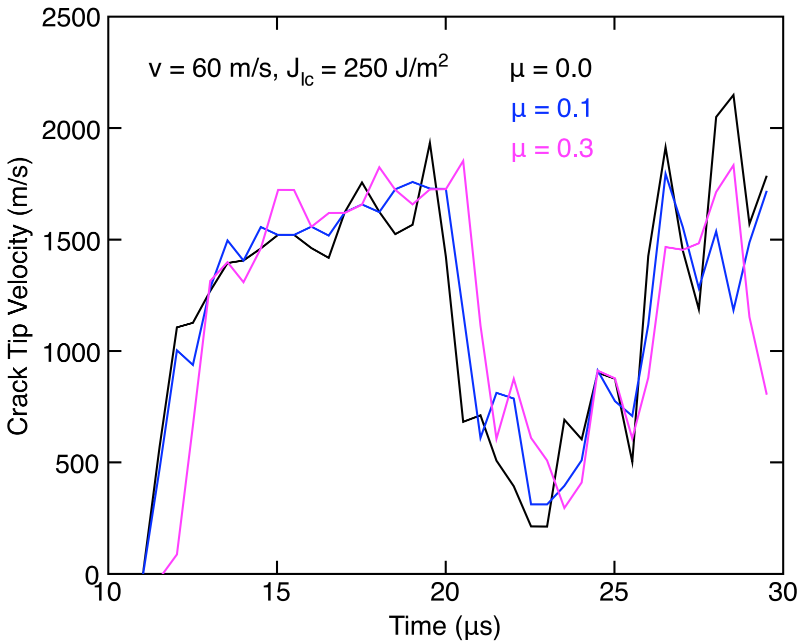
Experiment

J-Integral Fracture Simulations

Effect of friction ($\mu = 0.0$, $\mu = 0.1$, $\mu = 0.3$)

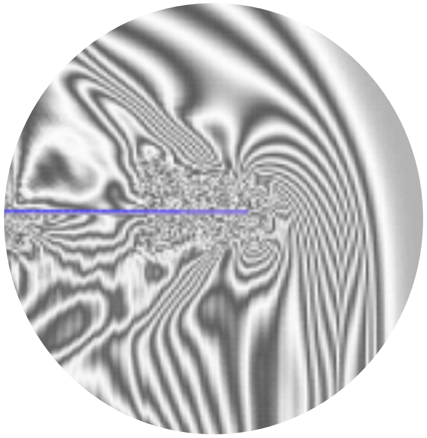
- Most dramatic for slowest crack tip propagation velocities ($J_{Ic} = 250$ J/m^2).
- For low impact velocities effect is strong
  - Delayed fracture propagation, slower crack tip velocity
- For higher impact velocities effect much weaker



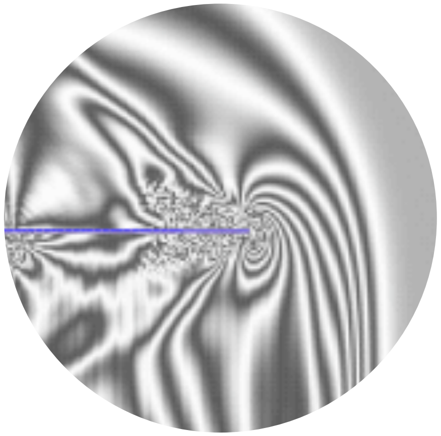


J-Integral Fracture Simulations

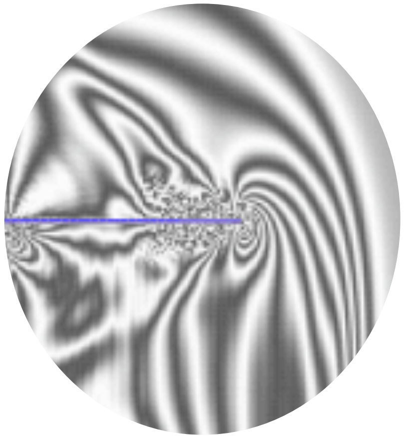
- v = 25 m/s, J_{Ic} = 250 J/m^2
- Fringe patterns suggest that friction on crack surface important (best fit to fringe pattern for μ = 0.3?



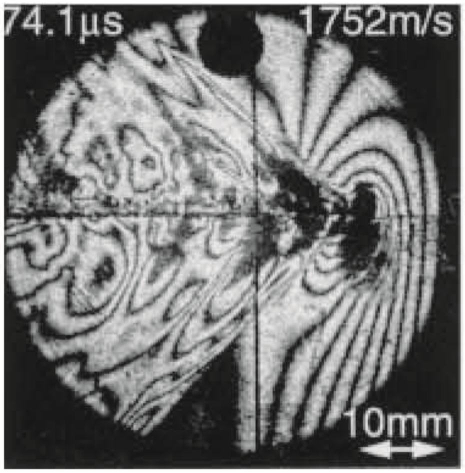
$\mu = 0.0$



$\mu = 0.1$



$\mu = 0.3$



Experiment

Conclusions

Fracture simulations using J-integral approach very encouraging:

- In good general agreement with previous results using cohesive zones
- Require far less numerical resolution (substantially more computationally efficient)
  - Longest runs presented here: < 3 hr.s, serial
  - Daphalapurkar, AMR: ~48 hr.s, parallel?
- Allow incorporation of frictional sliding on crack surfaces which also appears to be important.
- Allow incorporation of decohesion modeling in the crack, behind the tip (process zone), a very general modeling approach.