Application of a Quasi-Static Material Point Method in Geomechanics

GEO-INSTALL Modelling Installation Effects in Geotechnical Engineering

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August 9, 2010
1. Quasi-Static MPM

2. Moving Block

3. Soil-Structure Interaction

4. Cone Penetration in Undrained Clay

5. Outlook
1. Quasi-Static MPM

Quasi-static MPM & Updated Lagrangian FEM with implicit integration for load step $i$

Subincremental displacement vector

Load step number

$K^i \delta u^i_{k+1} = F^i_{\text{external},k} - F^i_{\text{internal},k}$

Iteration number

Elastic stiffness matrix

Load

$\Delta F^i$

$\Delta u^i = \sum_{k=1}^{\Delta u^i} \delta u^i_k$

Steps

Displacement of point A

A
Particles represent the deforming solid body inside a finite element mesh.

1. Quasi-Static MPM

- Beginning of load step
- Mesh distortion during load step
- Resetting at end of load step
2. Moving Block

15-noded prismatic element with near-quadratic interpolation
2. Moving Block

Active block movement

Elastic block

\[ E = 100 \text{ MPa}, \quad \nu = 0.0 \]
\[ \gamma = 20 \text{ kN/m}^3 \]

Full bonding with soil

Soil (Mohr-Coulomb)

\[ E = 10 \text{ MPa}, \quad \nu = 0.3 \]
\[ c = 10 \text{ kPa}, \quad \varphi = 30^\circ \]
\[ \gamma = 20 \text{ kN/m}^3 \]
2. Moving Block

Active block movement

Shift of block = 0 cm
2. Moving Block

Active block movement

Shift of block = 5 cm
2. Moving Block

Active block movement

Shift of block = 10 cm
2. Moving Block

Active block movement

Shift of block = 15 cm
Shift of block = 20 cm
The block is loosing its contact to the soil, a free slope is forming. Soil is slightly heaving up in front of the block.

Shift of block = 25 cm
The block is loosing its contact to the soil, a free slope is forming. Soil is slightly heaving up in front of the block.

Shift of block = 30 cm
The block is losing its contact to the soil, a free slope is forming. Soil is slightly heaving up in front of the block.

Shift of block = 35 cm
A few soil particles are still sticking to the block, due to soil cohesion and adhesion between soil and block.

Shift of block = 40 cm
2. Moving Block

Passive block movement

Shift of block = 15 cm
2. Moving Block

Passive block movement

Shift of block = 22.5 cm
2. Moving Block

Passive block movement

Shift of block = 30 cm
2. Moving Block

Passive block movement

Shift of block = 37.5 cm
2. Moving Block

Passive block movement

Shift of block = 45 cm
2. Moving Block

Passive block movement

Shift of block = 52.5 cm
2. Moving Block

Passive block movement

Shift of block = 60 cm

End of movement
2. Moving Block

Passive block movement – reaction forces

Horizontal Equilibrium

Prescribed displacement \( u_x \) [mm]
3. Soil-Structure Interaction
3. Soil-Structure Interaction

Application of interface elements with the MPM

Interface tractions: \[ \dot{t} = D \dot{w} \] with \[ \dot{w} = \dot{u}_{\text{face } A} - \dot{u}_{\text{face } B} \]

Elastic-plastic stiffness matrix:

\[
D = D^e - \frac{1}{d} D^e \frac{\partial g}{\partial \dot{t}} \frac{\partial f^T}{\partial \dot{t}} D^e
\]

\[ f(t) \] ... yield function \quad \[ g(t) \] ... plastic potential function

\[ d = \frac{\partial f^T}{\partial \dot{t}} D^e \frac{\partial g}{\partial \dot{t}} \]
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Activated volume elements contain particles which carry the stresses.

Activated interface elements have no particles!

New interface stresses need to be computed after mesh resetting.
Supervector of external nodal forces: \[ \mathbf{F}_{\text{external}}^{\text{interface}} = \int_{V_{\text{soil}}} \mathbf{B}^T \boldsymbol{\sigma} \, dV + \int_{V_{\text{soil}}} \mathbf{N}^T \gamma \, dV \]

\( \mathbf{B} \) = strain interpolation matrix \quad \boldsymbol{\sigma} = \text{soil stresses} \n
\( \mathbf{N} \) = shape function matrix \quad \gamma = \text{soil weight} \n
Supervector of internal nodal forces: \[ \mathbf{F}_{\text{internal}}^{\text{interface}} = \int_{S_{\text{interface}}} \mathbf{N}^T \mathbf{t} \, dS \]

Unknown tractions are obtained through solving system of equilibrium equations for \( \mathbf{t} \) at (Newton-Cotes) integration points.
3. Soil-Structure Interaction

Sliding of elastic block

adhesion = 10 kPa
3. Soil-Structure Interaction

Sliding of elastic block

Low-order

High-order

\[ \sigma_N [\text{kPa}] \]

\[ \tau_s [\text{kPa}] \]

\[ x [\text{m}] \]

\[ x [\text{m}] \]

\[ x [\text{m}] \]

\[ x [\text{m}] \]
4. Cone Penetration in Undrained Clay

4-noded tetrahedral element with linear interpolation
4. Cone Penetration in Undrained Clay

In-situ site investigation with cone penetration test
4. Cone Penetration in Undrained Clay

Discretisation of Cone Penetrometer

\[ E_U = 6 \text{ MPa} \quad v_U = 0.40 \quad c_U = 20 \text{ kPa} \quad \varphi_U = 0^\circ \]

Segment discretised with 4-noded tetrahedral elements
4. Cone Penetration in Undrained Clay

Results

Rough contact: adhesion = $c_u$

Adhesion = $c_u / 2$

Smooth contact: adhesion = 0
Rough contact

Normal stresses

Shear stresses

4. Cone Penetration in Undrained Clay

Results

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4. Cone Penetration in Undrained Clay

Results

Principal stresses

Smooth

Rough
4. Cone Penetration in Undrained Clay

Results

Rough contact at 4 D

Incremental horizontal displacements
4. Cone Penetration in Undrained Clay

Results

Accumulated shear strain at 4 D

Rough contact

Smooth contact
5. Outlook
5. Outlook

- Pore pressure dissipation
- Going beyond Mohr-Coulomb (Hardening)
- Get experience in layered soil
- In future also interaction between piles

1 September 2010 MPM Workshop Deltares, Delft, The Netherlands
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### Workshop on Geotechnical Installation

**13 september 2010**

**Introduction**

This workshop is jointly organised by the Geo-Install consortium and the Plant Development Community. The Geo-Install consortium concentrates on the modelling of installation effects in geotechnical engineering, which includes the development of advanced material and computational models. Geo-Install consists of the universities of Delft, Delft University of Technology, Delft, Delft University of Technology, Delft, Delft University of Technology, and the Norwegian University of Science and Technology. The consortium is supported by funding from the European Commission. The Plant Development Community is a consortium of companies that focuses on the development of the Flora software for the analysis of geotechnical structures.

The aim of this workshop is to present the state-of-the-art on constitutive modelling as well as on the numerical simulation of large-deformation processes in geotechnical engineering. In order to do so, specialists from computational geotechnics and applied geotechnical engineering will lecture during this workshop.

For organisational purposes, all participants are required to register before 15 August 2010 and pay the workshop fee for lunch etc. of 30 € in advance. The website www.deltatech.nl is recommended for those who need accommodation. You will have to book early, as Delft is a tourist city.

**Venue of Congress**

Delft, Steenweg 2, 2600 MH Delft, The Netherlands
E-Mail: turbantaz@deltares.nl

**Agenda**

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<td>09.00</td>
<td>Coffee</td>
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<td></td>
<td>09.45</td>
<td>Opening Prof. Peter Verman / University of Stuttgart &amp; Delft</td>
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**Morning Session**

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<td>10.00</td>
<td>Dr. Lars Andersen</td>
<td>Large deformation / Installation effects Norwegian Geotechnical Institute</td>
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<tr>
<td>10.15</td>
<td>Dr. Mineo Kanasako</td>
<td>Mechanical behaviour of clay</td>
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<td>10.30</td>
<td>Prof. Dietrich Osterberg</td>
<td>Mechanical behaviour of sand</td>
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<td>11.00</td>
<td>Prof. Bernhard Moller</td>
<td>State of the art in experimental research</td>
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<td>11.15</td>
<td>Winter</td>
<td>State of the art in numerical modelling</td>
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<td>11.30</td>
<td>Prof. Frank Moller</td>
<td>Theoretical models</td>
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<td>11.45</td>
<td>Prof. Michael Heeks</td>
<td>Modelling soil heterogeneity</td>
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<td>12.00</td>
<td>Prof. Peter Verman</td>
<td>Lunch</td>
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**Afternoon Session**

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<tr>
<td>13.30</td>
<td>Prof. Pieter Tijl</td>
<td>Driven pile behaviour</td>
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<td>14.00</td>
<td>Prof. Christian Moormann</td>
<td>Bored pile behaviour</td>
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<td>14.15</td>
<td>Dr. Jelle Olijf</td>
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<tr>
<td>14.30</td>
<td>Dietrich Osterberg</td>
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**13 augustus 2010**

**Deltatech**
Rough contact at 4 D

Incremental vertical displacements
\[ N_c = \frac{q_c}{c_u} \]

Range of results by Van den Berg

\[ I_r = \frac{G}{c_u} \]

Appendix


