A Few MPM Verification Problems Using Uintah

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August 9-10, 2010
6th MPM Workshop, Albuquerque, New Mexico

Outline

- Motivation
- Scope
- Verification Problems
- Acknowledgements

Motivation

- Applications in direct numerical simulations (DNS) of random heterogeneous materials can result in emergent behavior
- Improve acceptance of MPM to the numerics community

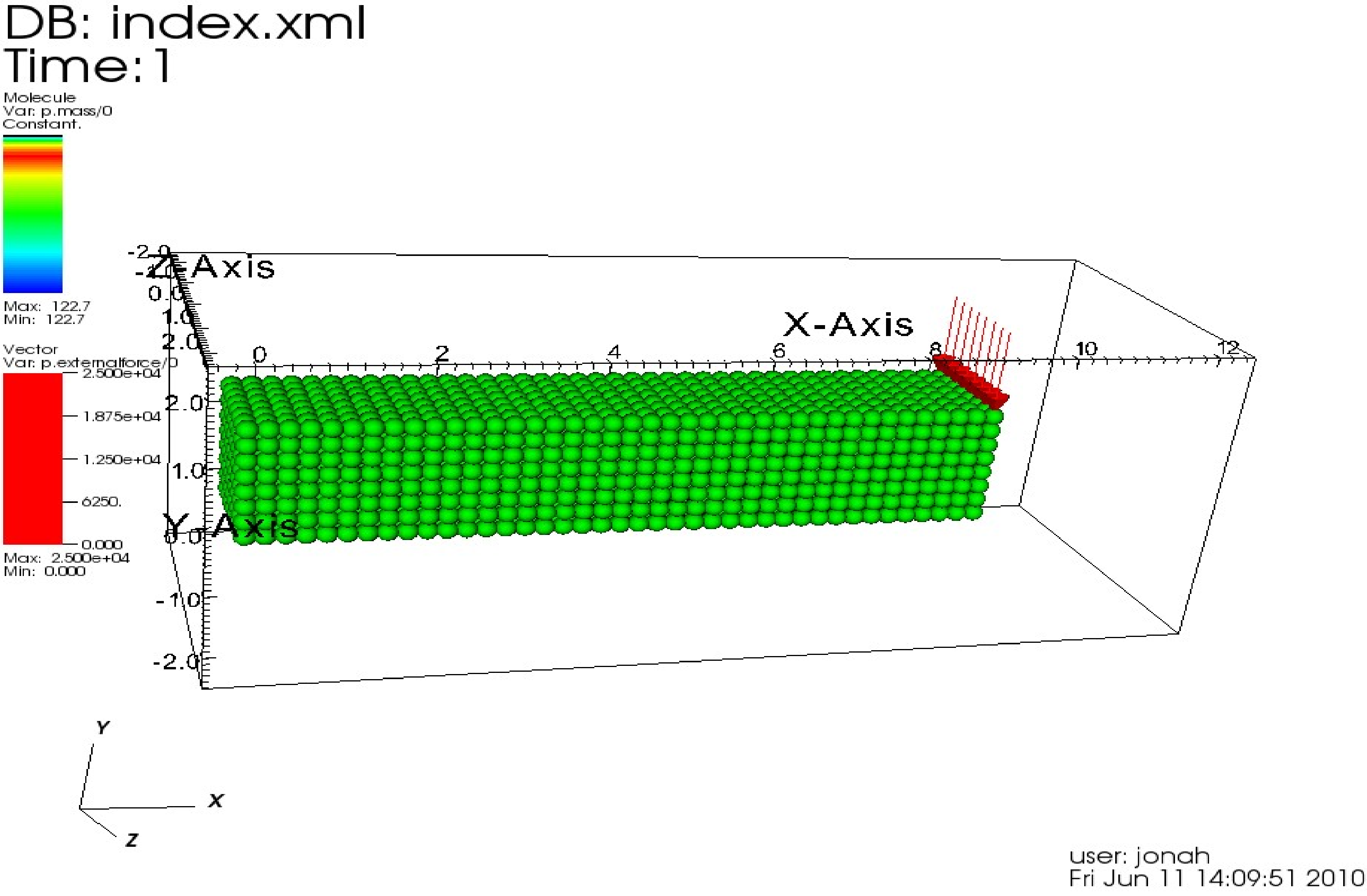
Scope

- Point of view of a user:
  - Should one use MPM for the problem at hand?
- Text-book problems
- ~10% error acceptable
- Efficiency not considered
- No fix attempted
- Uintah-MPM only (not ICE or MPMICE)

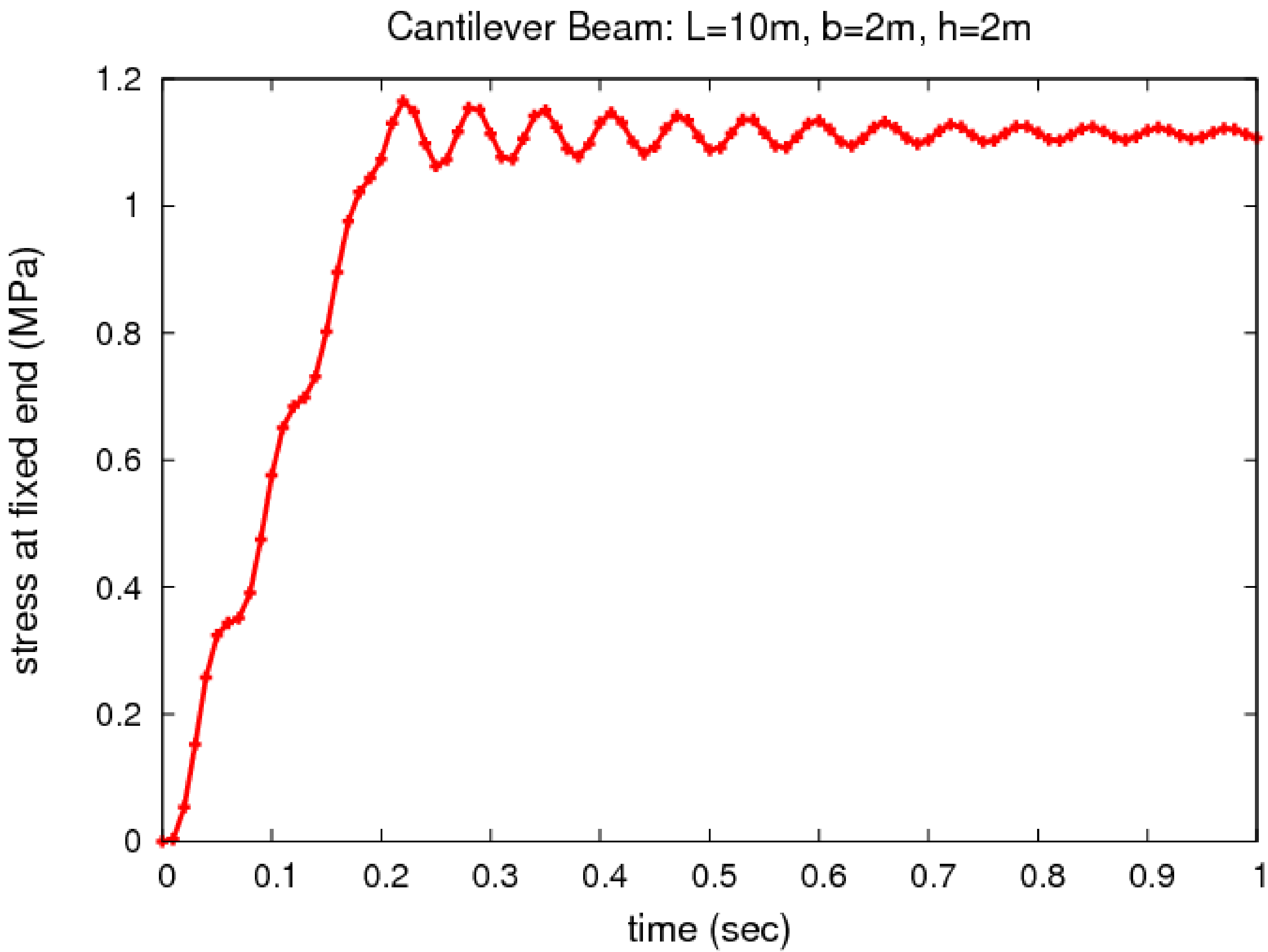
Verification Problems Using GIMP

- Cantilever beam
- Bar under gravity
- Fluid statics
- Rayleigh's problem
- Poisuelle flow
- Lid-driven cavity flow
- Stokes problem

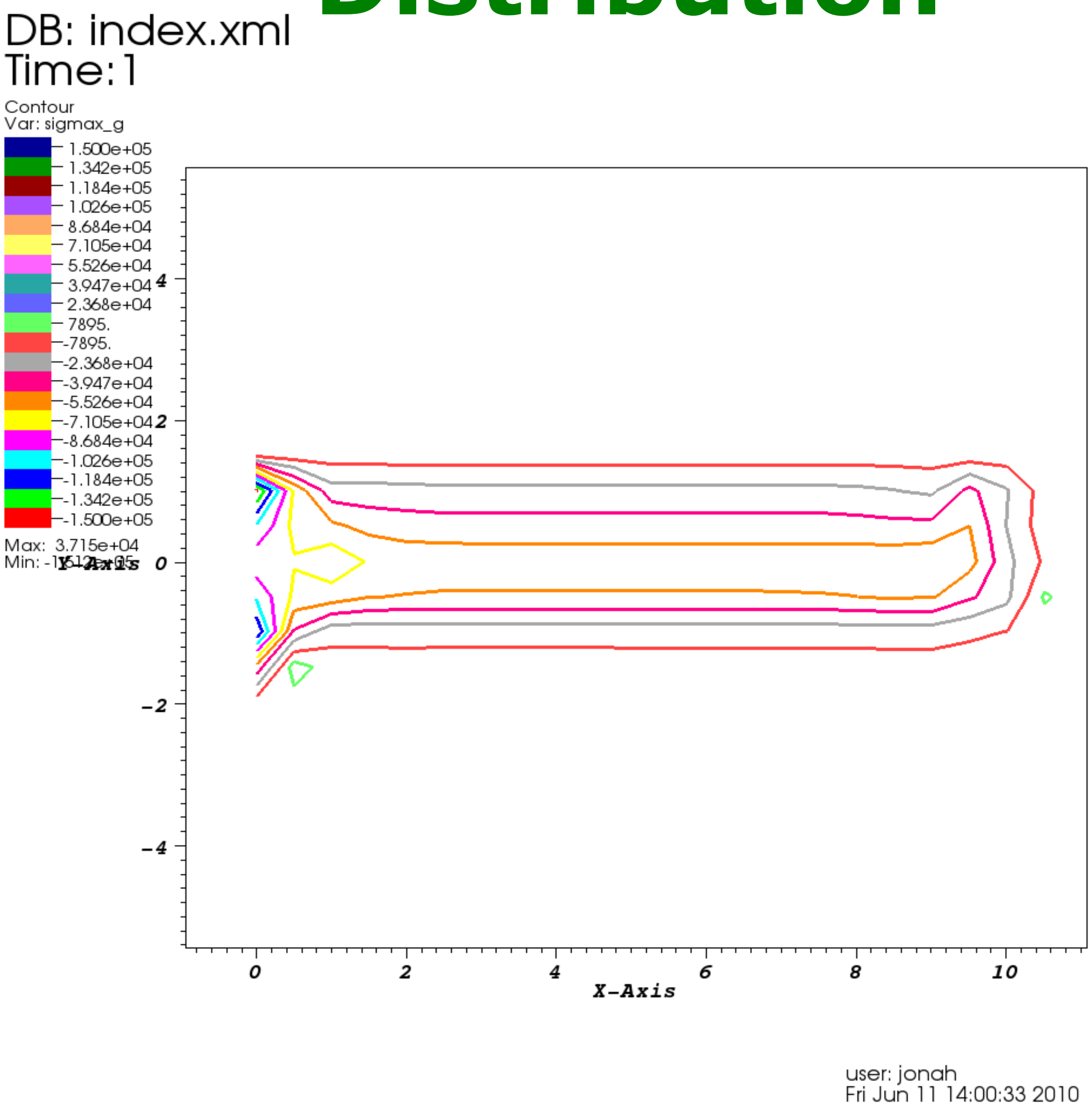
Cantilever Beam



Typical Time History



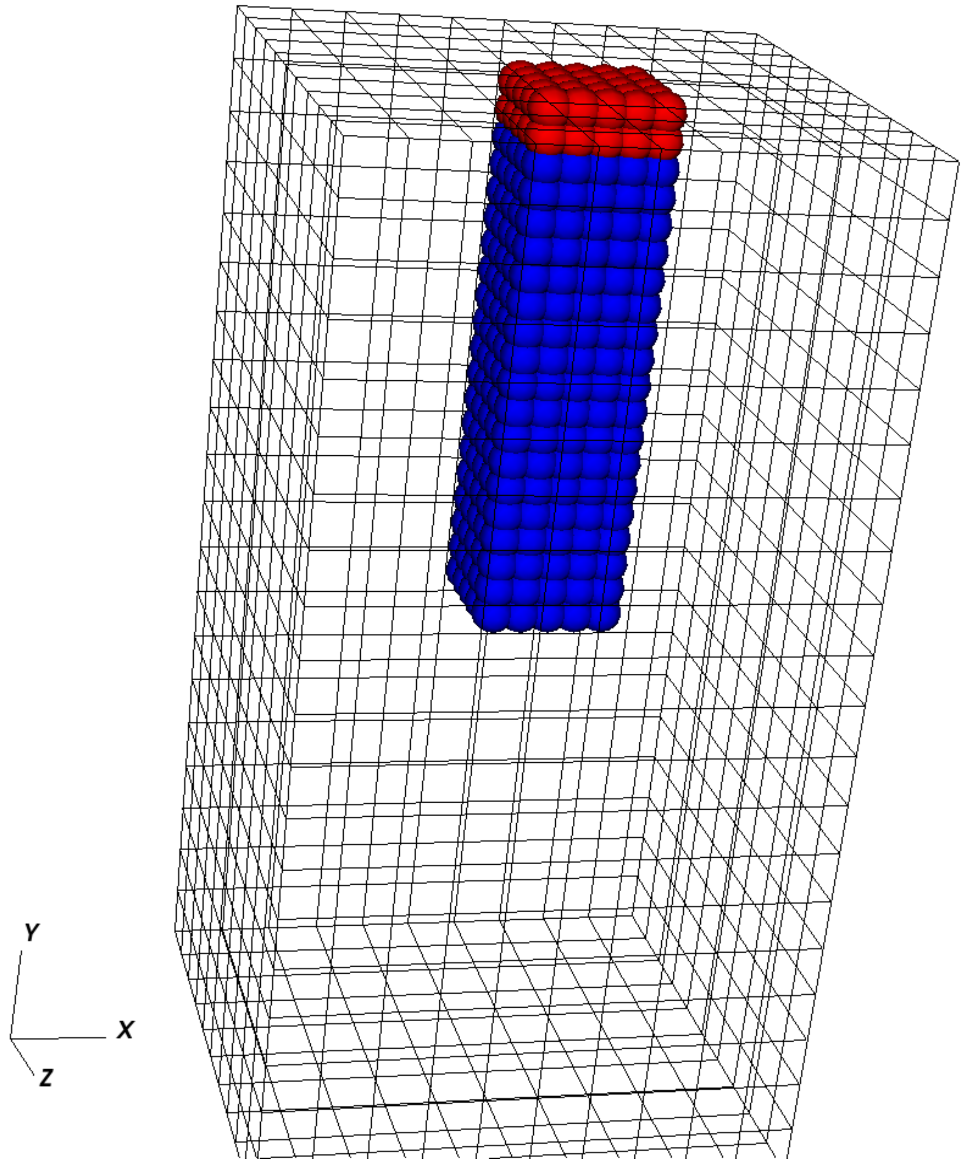
Typical Longitudinal Stress Distribution



Cantilever Beam Results

	PPC=1	PPC=2	PPC=3
Maximum Bending Stress/Theory	1.19	1.04	1.08
Maximum Shear Stress/Theory	0.89	0.83	0.84
Maximum Deflection/Theory	1	1.01	0.98

Bar Under Gravity



Theoretical Solution

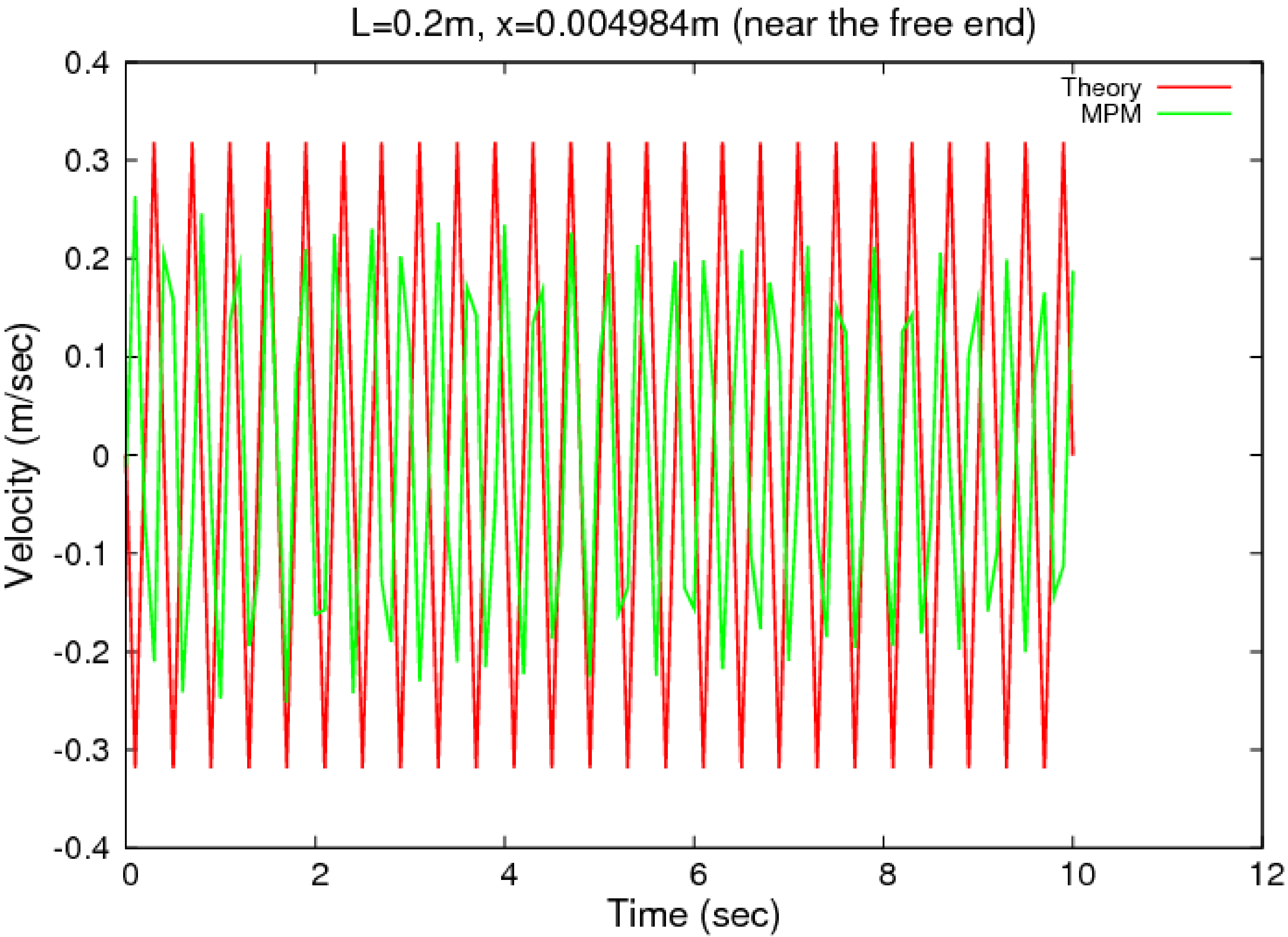
$$v(x,t) = gt - g\sum_{k=0}^{\infty}(-1)^k[H(t-\alpha_1)(t-\alpha_1) + H(t-\alpha_2)(t-\alpha_2)]$$

$$\sigma(x,t) = g\sqrt{E\rho}\sum_{k=0}^{\infty}(-1)^k[-H(t-\alpha_1)(t-\alpha_1) + H(t-\alpha_2)(t-\alpha_2)]$$

$$\alpha_1 = \frac{2kl + x}{c}, \alpha_2 = \frac{2l(k+1) - x}{c}$$

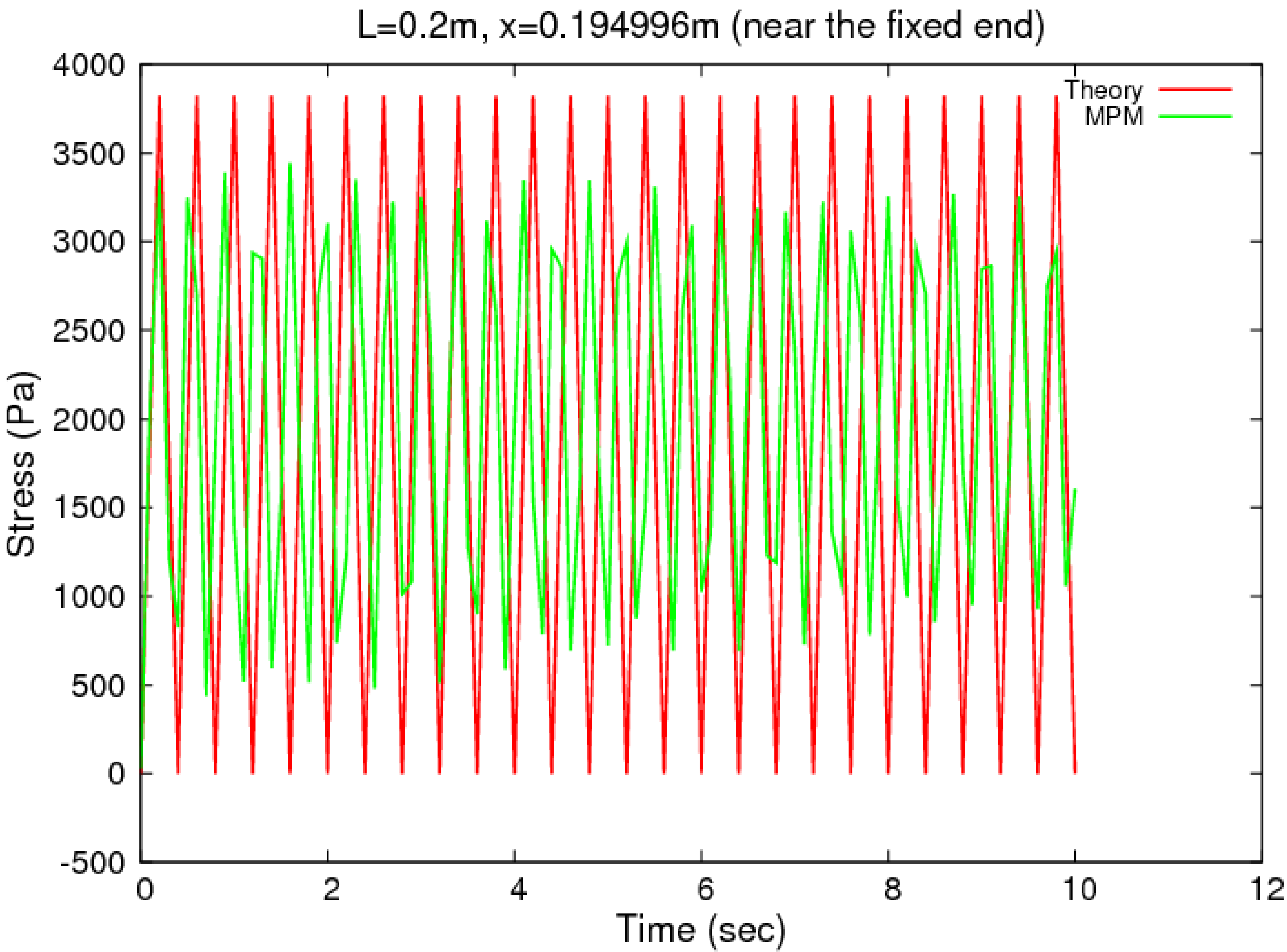
$$c = \sqrt{\frac{E}{\rho}}$$

Bar Under Gravity - Velocity

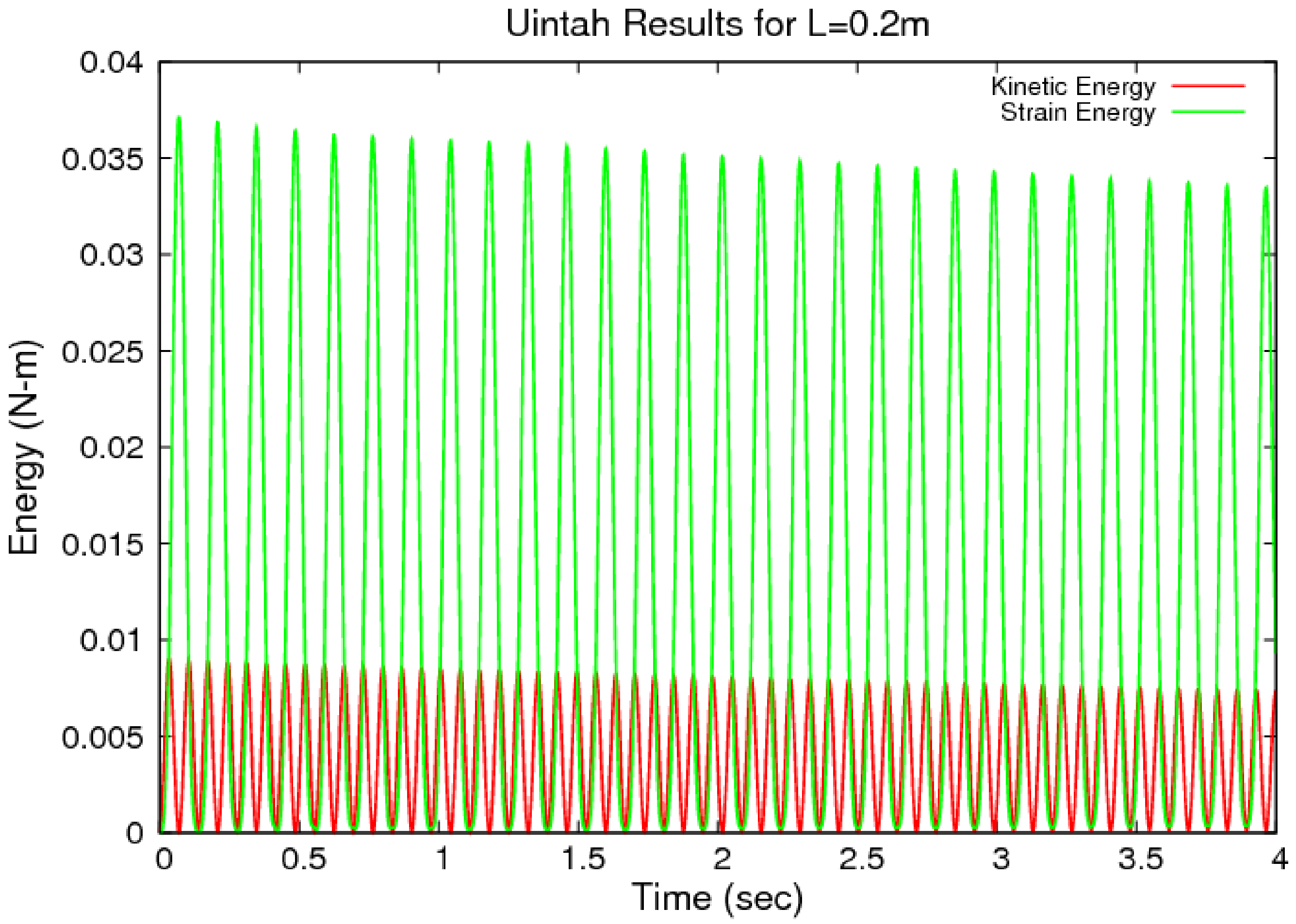


$$f_{\text{theory}} = 2.5 \text{ Hz}, f_{\text{MPM}} = 2.9 \text{ Hz}$$

Bar Under Gravity - Stress



Bar Under Gravity - Energy

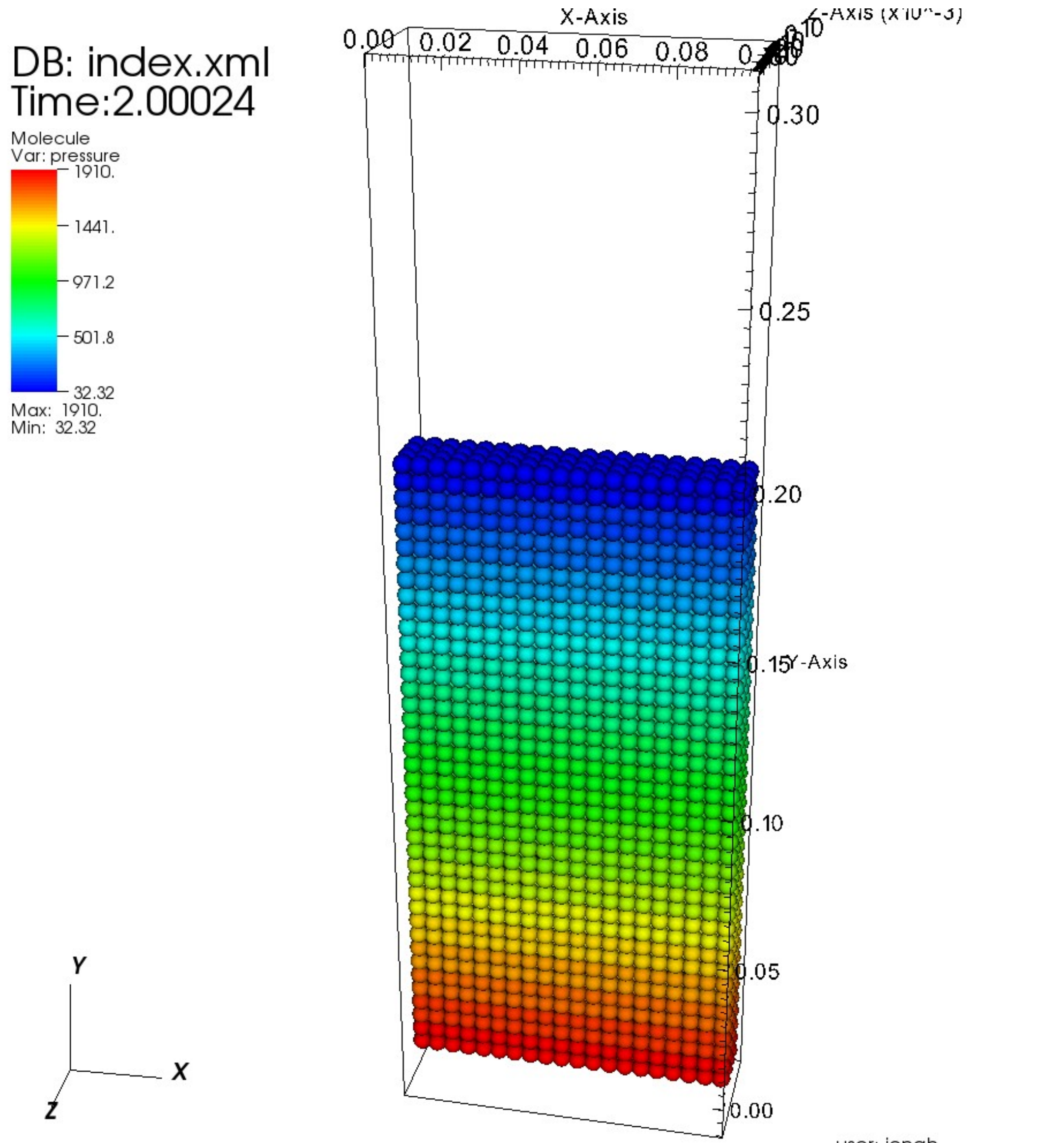


Nearly Incompressible Fluid

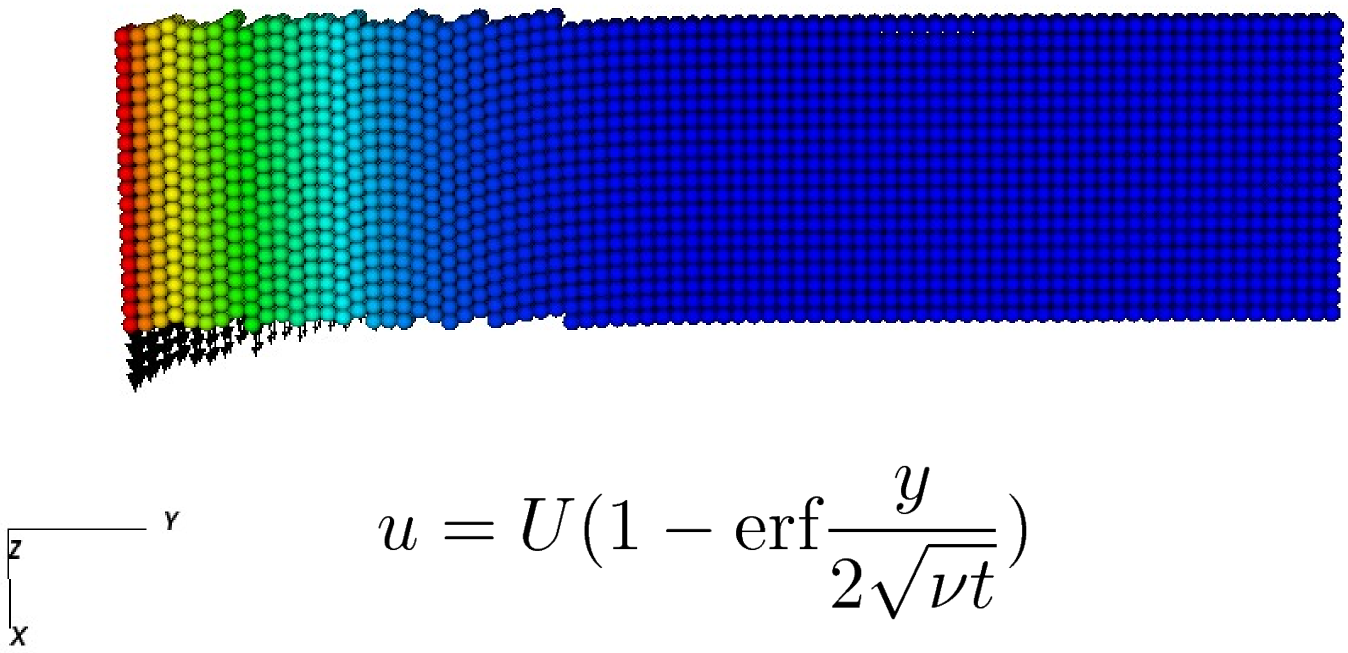
$$\sigma_{ij} = -p\delta_{ij} + 2\mu d_{ij}$$

$$p = k[(\frac{\rho}{\rho_0})^\gamma - 1]$$

Hydrostatic Stress (<1% error)



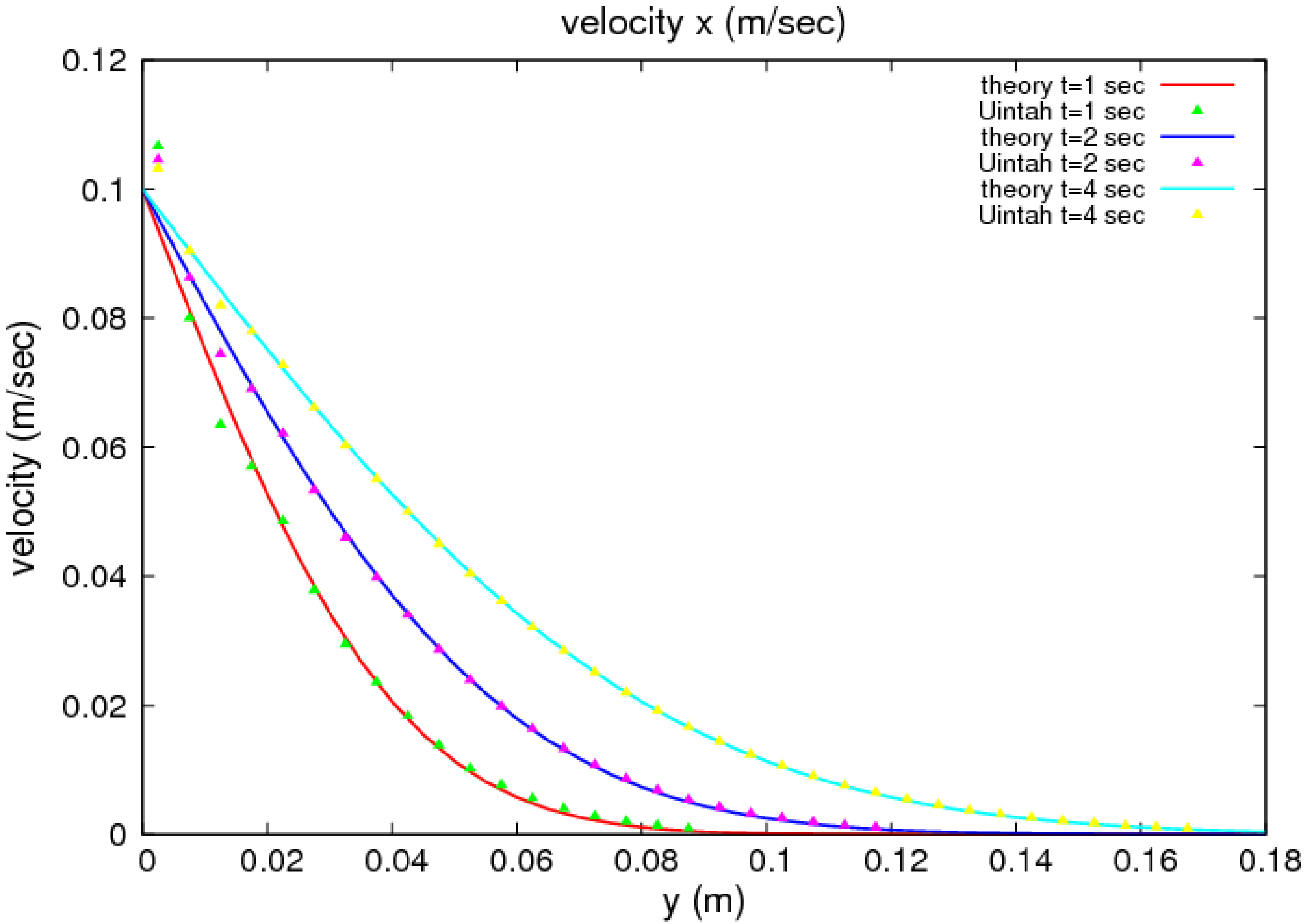
Rayleigh's Problem



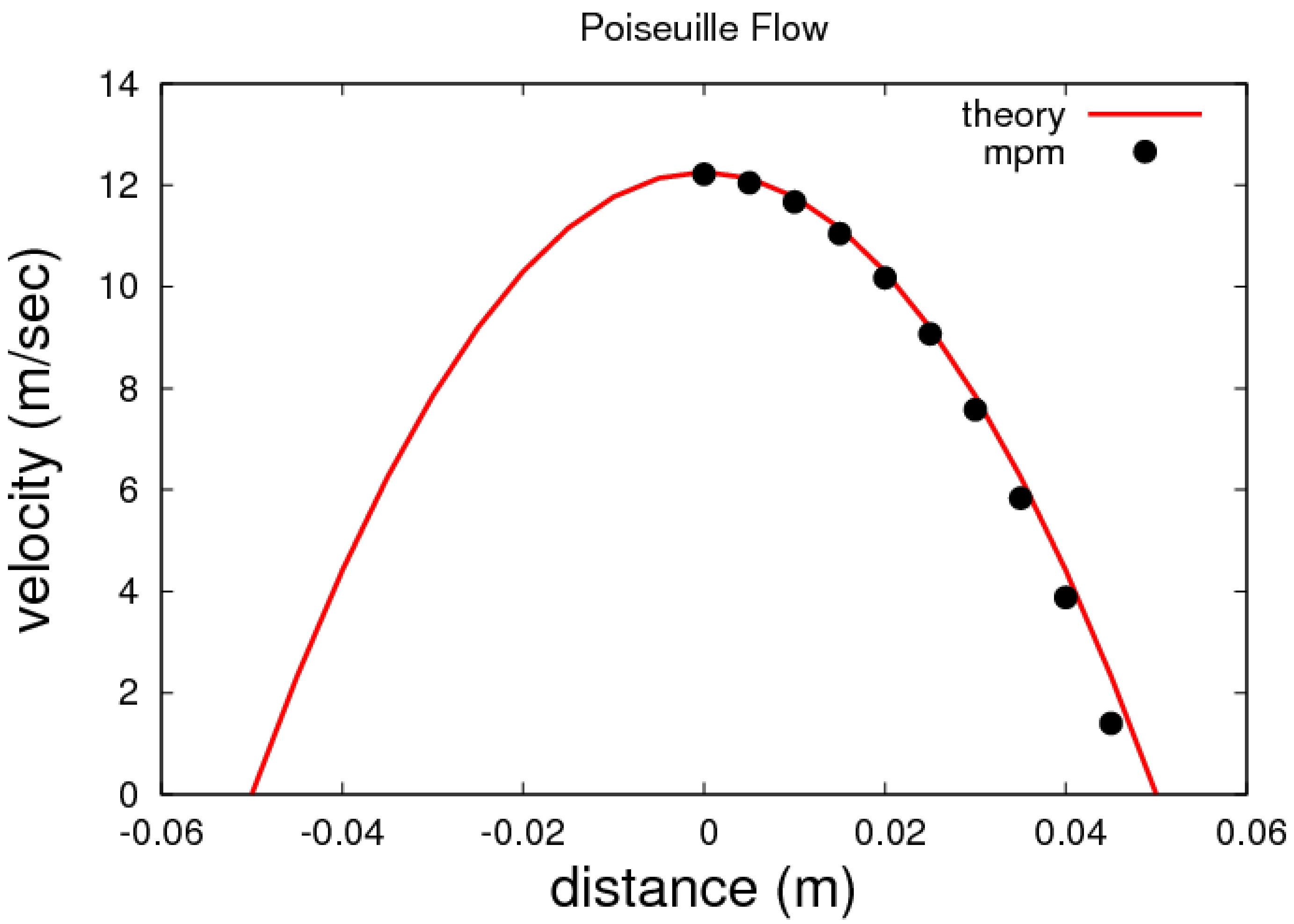
$$u = U\left(1 - \mathrm{erf}\frac{y}{2\sqrt{\nu t}}\right)$$

$$\nu = \frac{\mu}{\rho}$$

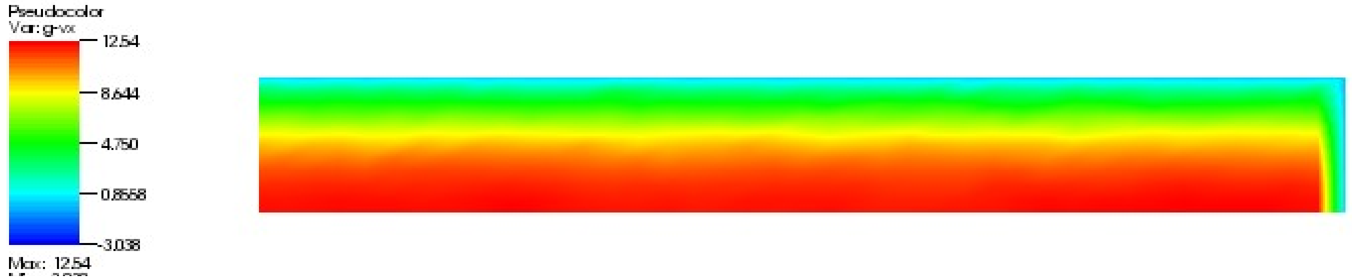
Rayleigh's Problem - Velocity

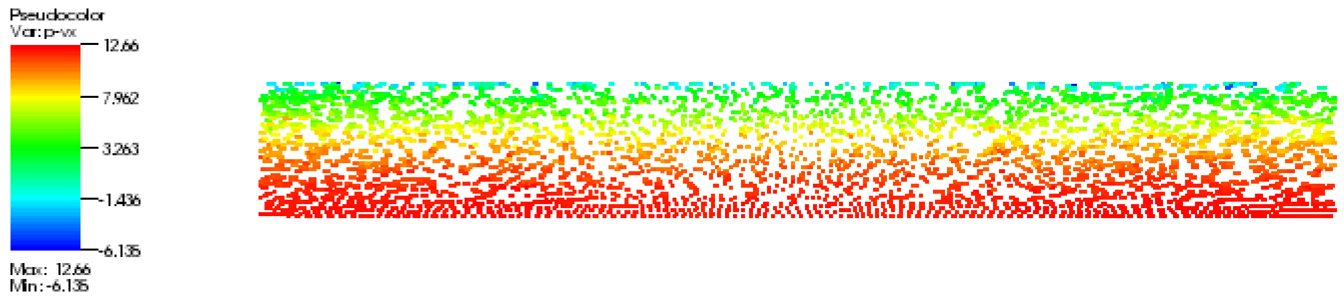


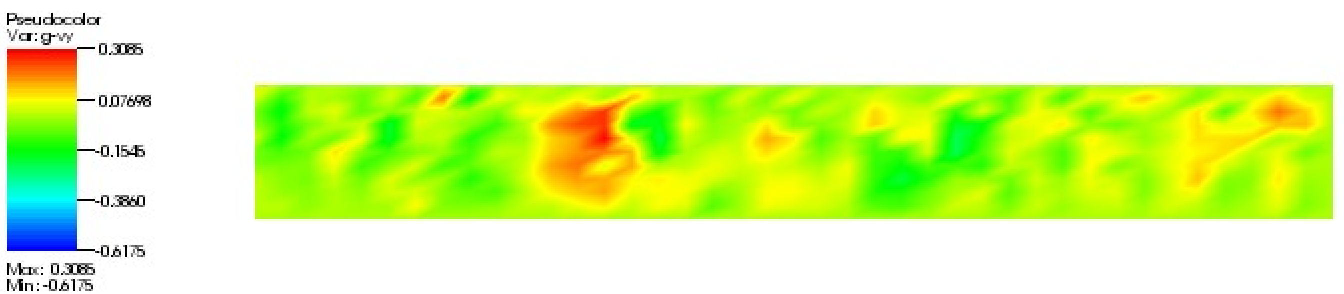
Poiseuille Flow (Re=1.0)
Gravity-driven; Periodic B.C.

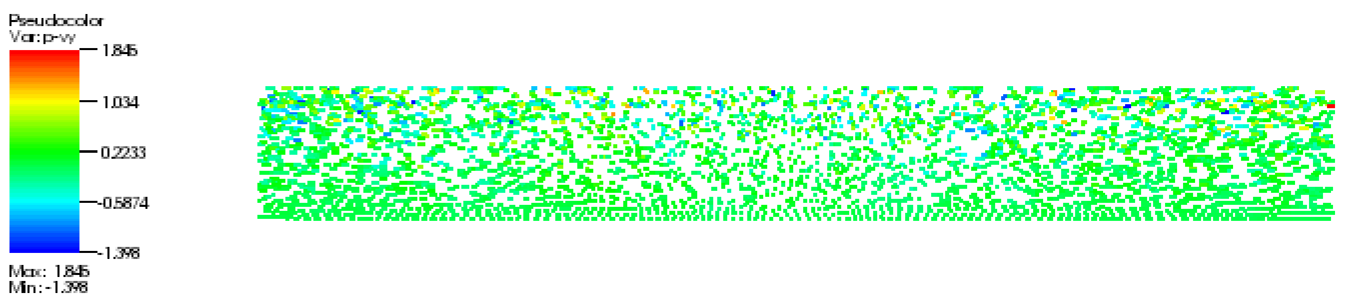


Poiseuille Flow-Numerical Instability

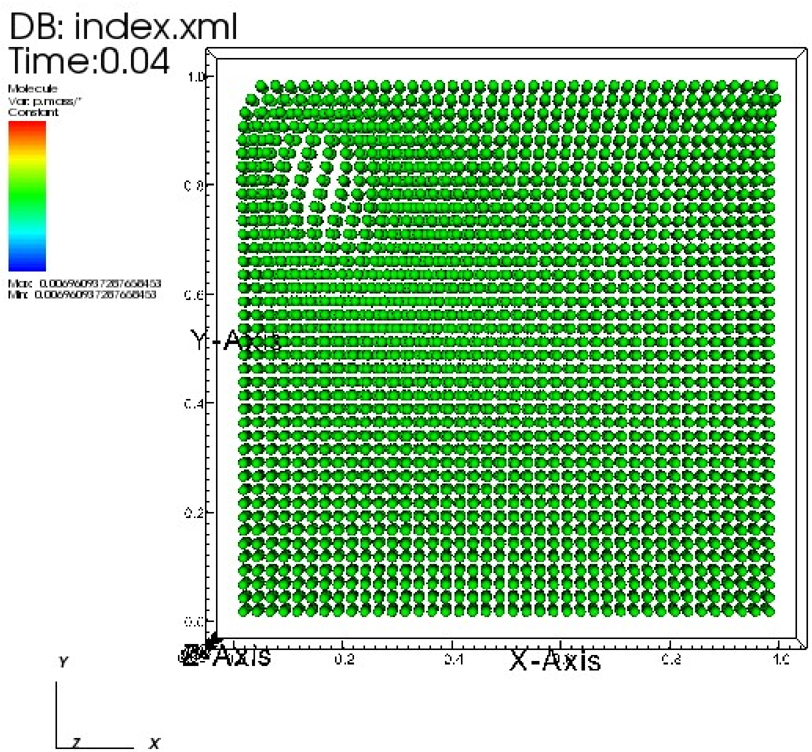


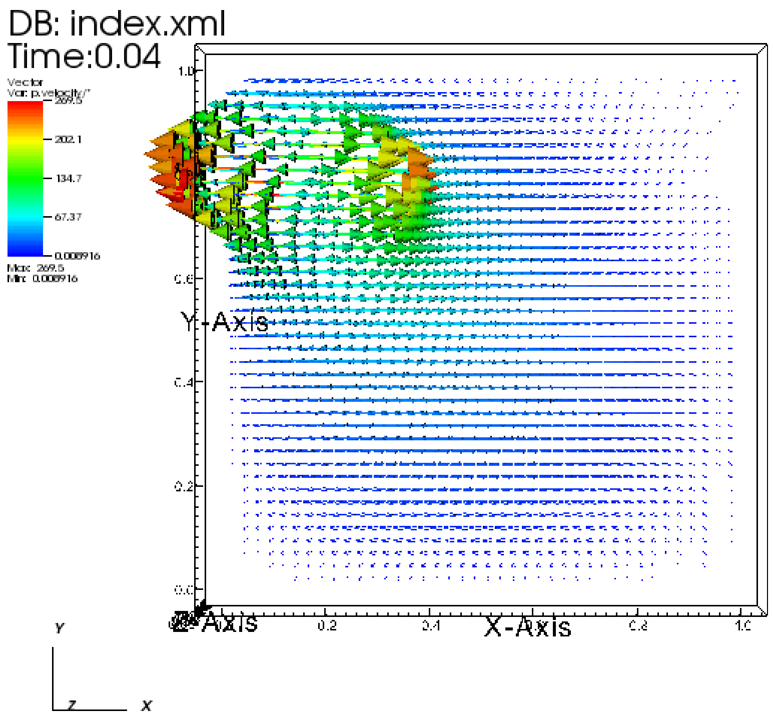






Lid-Driven Cavity Flow (Re=0.5) Possible?





Trivial problem for Uintah-ICE

Stokes Problem

- Rigid sphere falling in an infinite fluid
  - Sphere not deformable
  - No edge and bottom effects ($r/R < 0.1$)
- $Re << 1$ (creeping flow)
- Weight balanced by buoyancy and drag forces
- Terminal velocity used in viscometer

Stokes Problem - Theory

$$W = F_{\text{buoyancy}} + F_{\text{drag}}$$

$$W = \frac{4}{3}\pi r^3 \rho_{\text{solid}} g$$

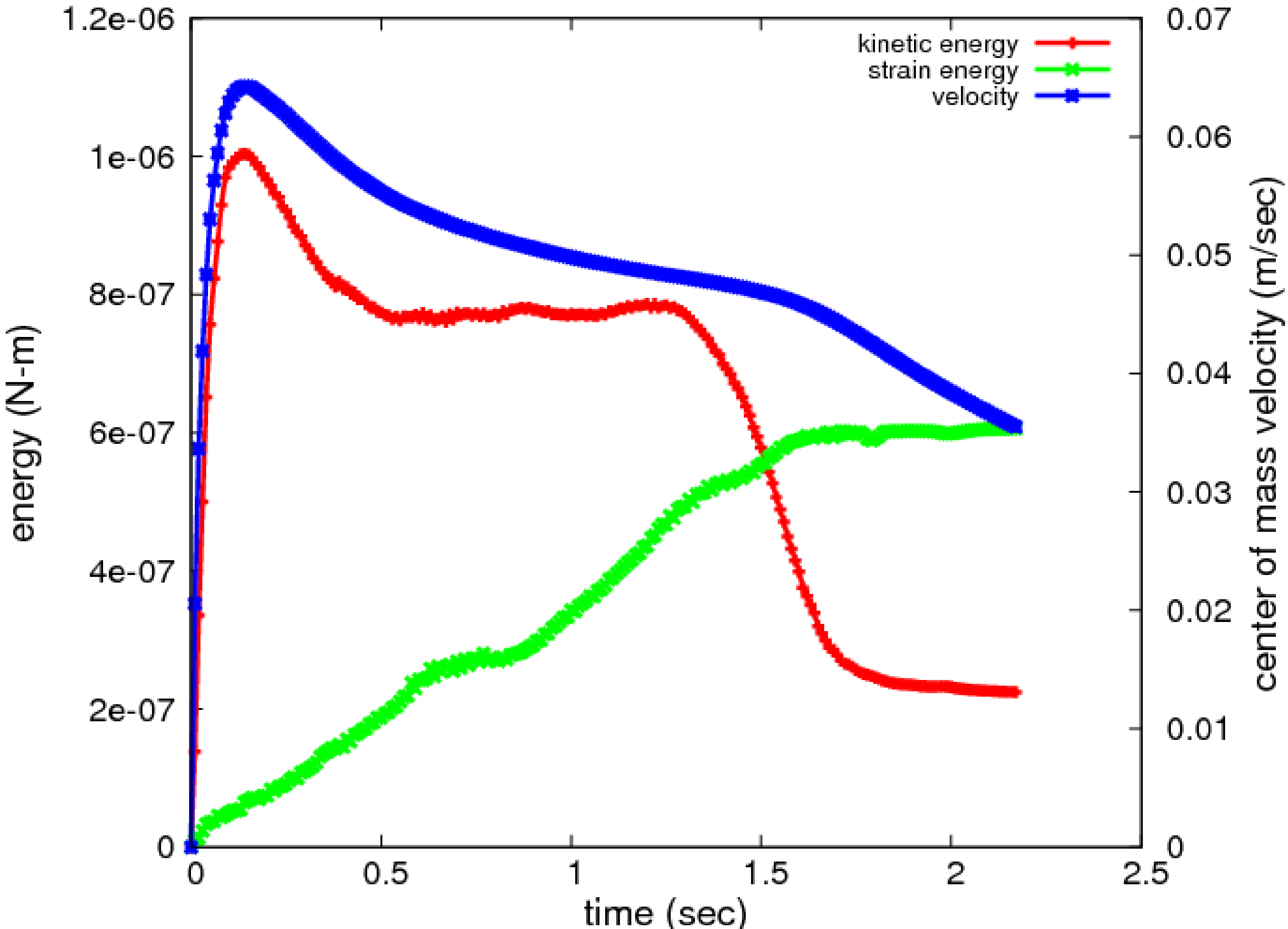
$$F_{\text{buoyancy}} = \frac{4}{3}\pi r^3 \rho_{\text{fluid}} g$$

$$F_{\text{drag}} = 6\pi\mu r v$$

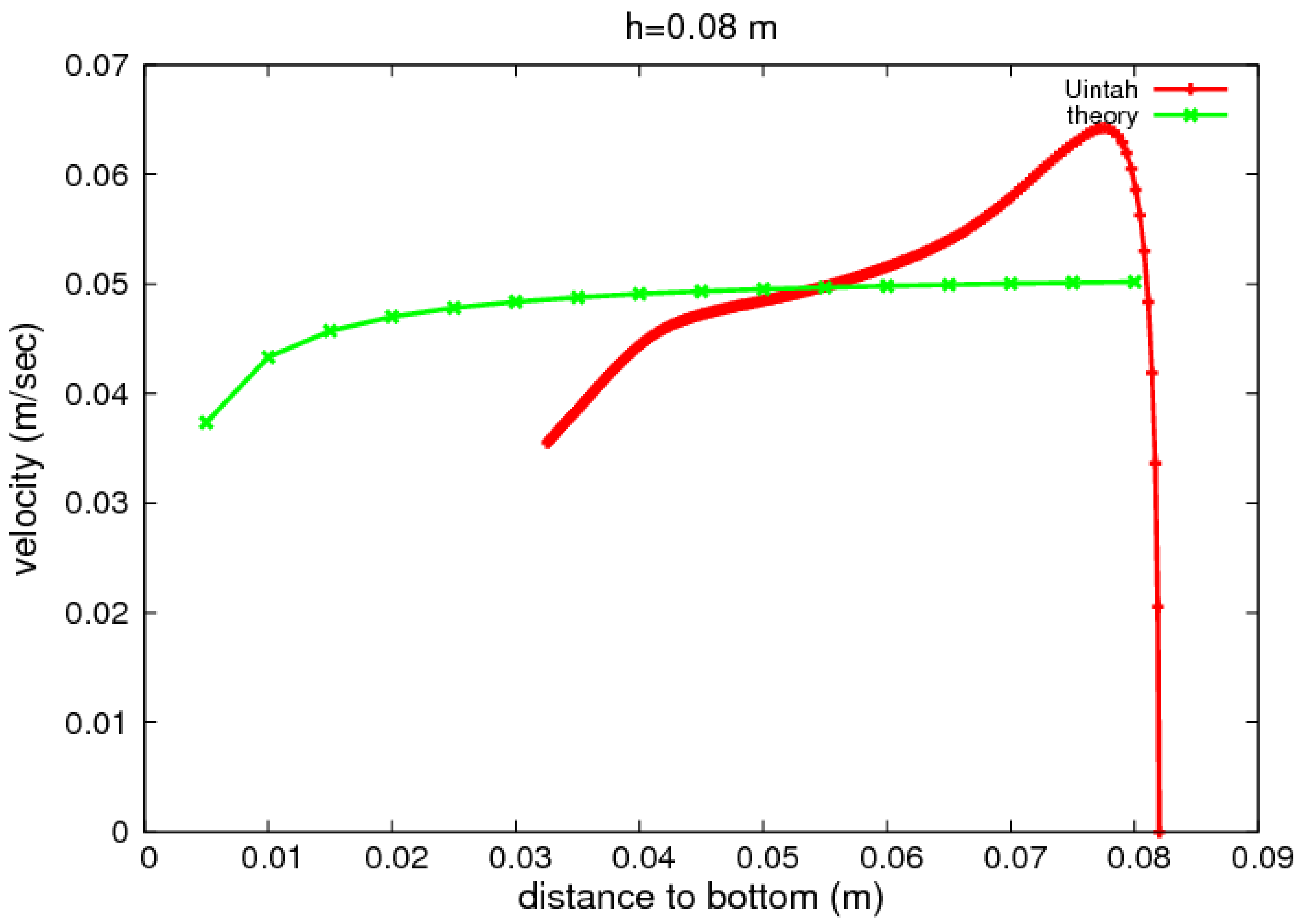
$$F_{\text{drag corrected}} = F_{\text{drag}} \times (1 + \frac{3}{16}\text{Re} + \frac{9}{8}\frac{r}{z})$$

$$\text{Re} = \frac{2\rho_{\text{fluid}} v r}{\mu}$$

Stokes Problem – Time History



Stokes Problem – MPM vs Theory



~1 million particles; 1,344 cpu hours

Acknowledgements

- Jim Guilkey and the Uintah team
- Professor Cheng-fu Chen at UAF
- Students in ME 693 2009 at UAF
- Arctic Region Supercomputing Center (ARSC) at UAF