Evaluation of a Smeared Crack Approach in MPM as a Computational Failure Method

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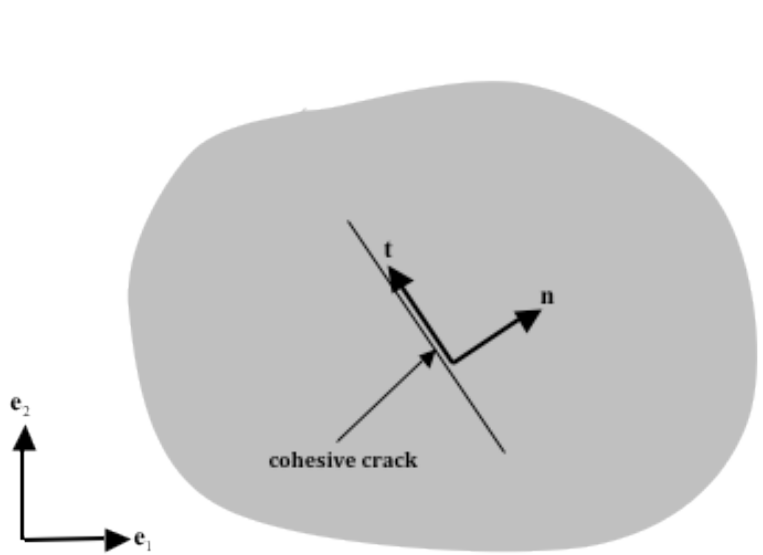
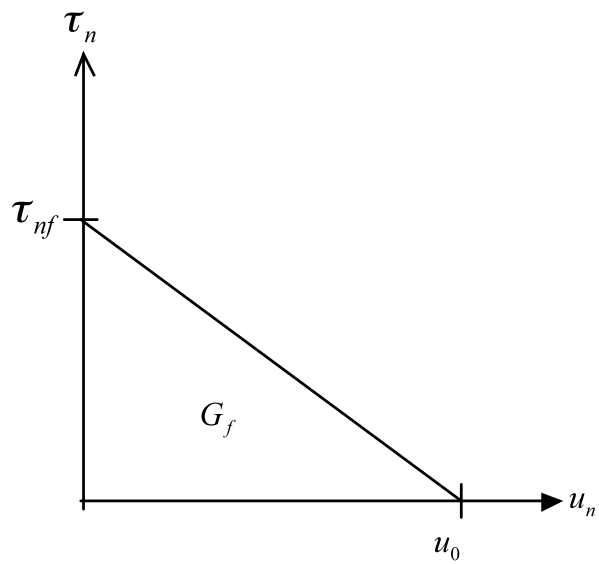
Motivation and Objective

- Smeared crack MPM used to model opening of leads (cracks) in sea ice motion [Sulsky et al., 2007], [Peterson, 2008]
- Assess limitations of smeared crack MPM as a computational fracture method
- Implicit dynamics MPM utilized [Sulsky and Kaul, 2004]

Discrete Constitutive Models for Material Failure

Represents FPZ in quasi-brittle failure such as concrete by relating traction to the jump in the displacement disontinuity across a failure surface [Hillerborg et al., 1976], [Schreyer, 2007]

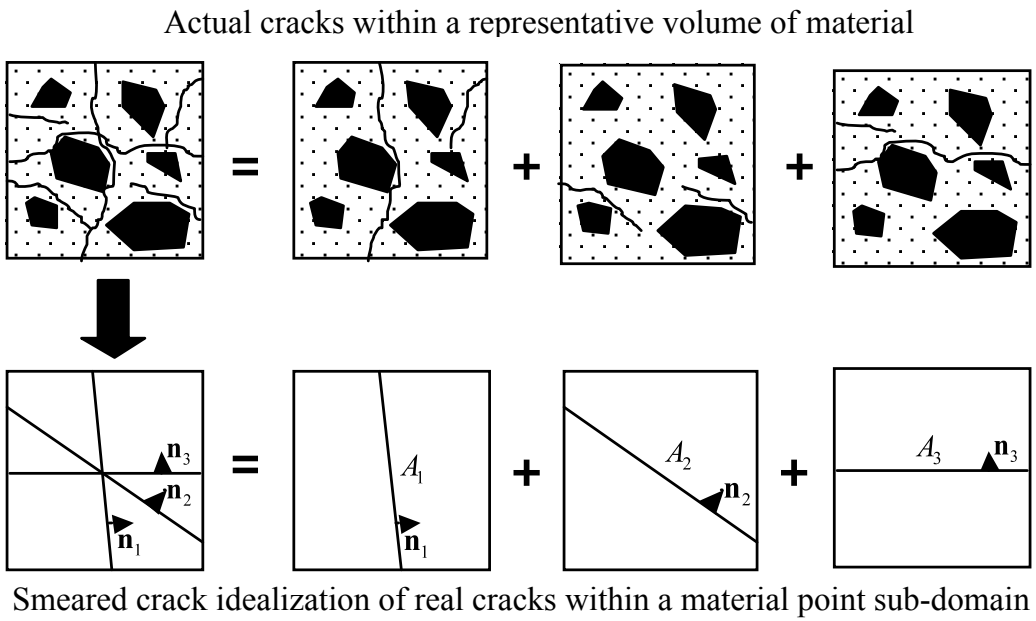




$\{\mathbf{n}, \mathbf{t}\}$	failure surface basis
$\boldsymbol{\tau} = \tau_n \mathbf{n} + \tau_t \mathbf{t}$	traction
$[[\mathbf{u}]] = [[u_n]]\mathbf{n} + [[u_t]]\mathbf{t}$	displacement discontinuity
G_f	fracture energy
τ_{nf}	ultimate tensile strength
u_0	critical crack opening

Smeared Crack Representation of Failure

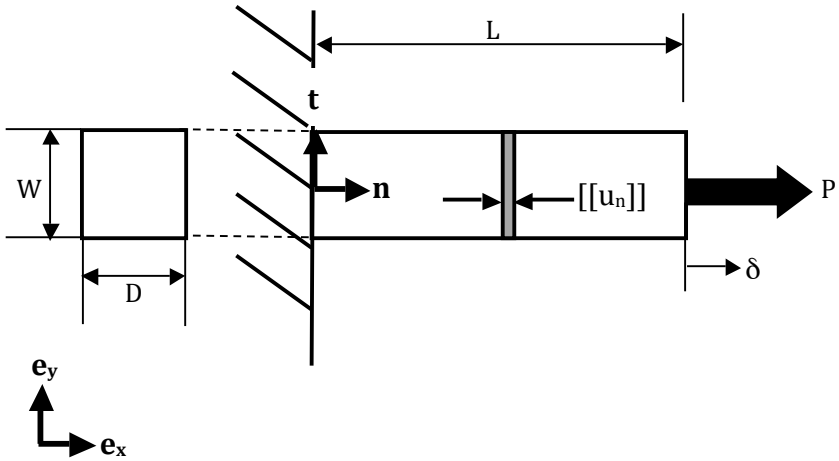
Smeared crack FEM: [Rots, 1988], [de Borst and Nauta, 1985], [Bazant and Oh, 1983]

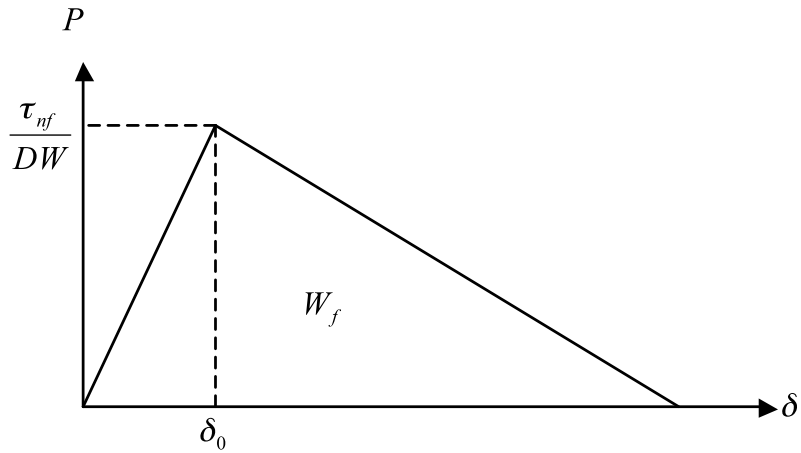


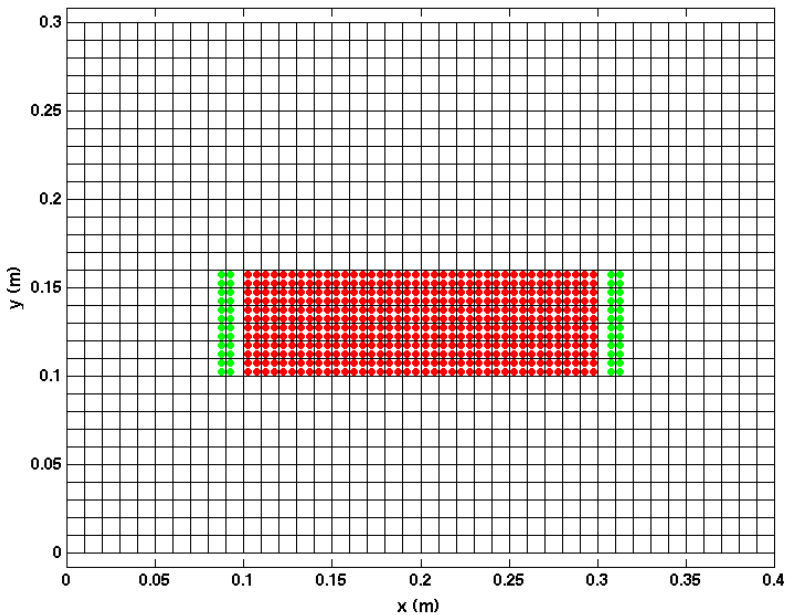
$$\underbrace{\varepsilon}_{\text{total strain}} = \underbrace{\varepsilon^e}_{\text{elastic strain}} + \underbrace{\varepsilon^{dc}}_{\text{de-cohesive strain}}$$

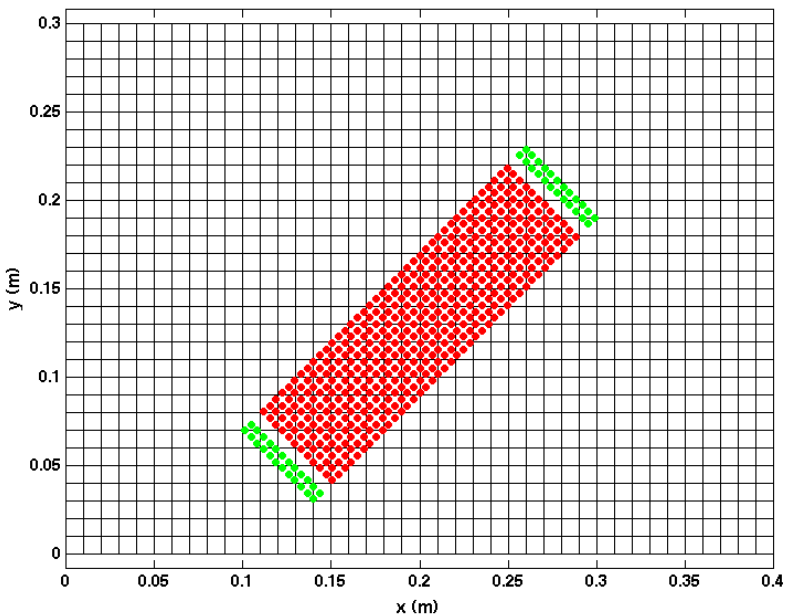
$$\varepsilon^{dc} = \sum_i \varepsilon_i^{dc}, \qquad \varepsilon_i^{dc} = \frac{1}{L_c} \left([[\mathbf{u}]]_i \otimes \mathbf{n}_i \right)^S, \qquad L_c \quad \text{characteristic length}$$

Case Study Problem: Elastic De-cohesive Tensile Bar



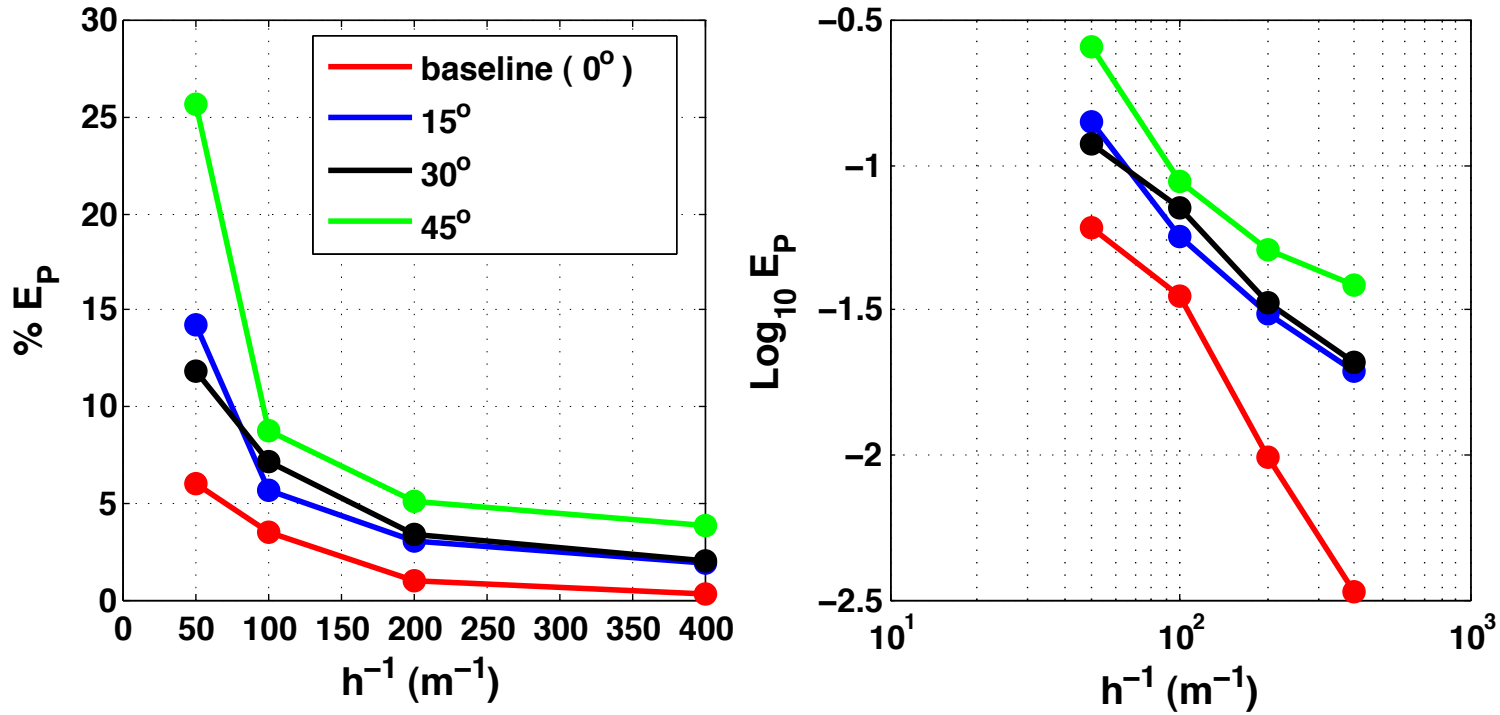






Preliminary Study:
Grid Orientation Bias for Elastic Tensile Bar

Rate of convergence affected for different $\alpha = \cos^{-1}(\mathbf{n} \cdot \mathbf{e}_x)$

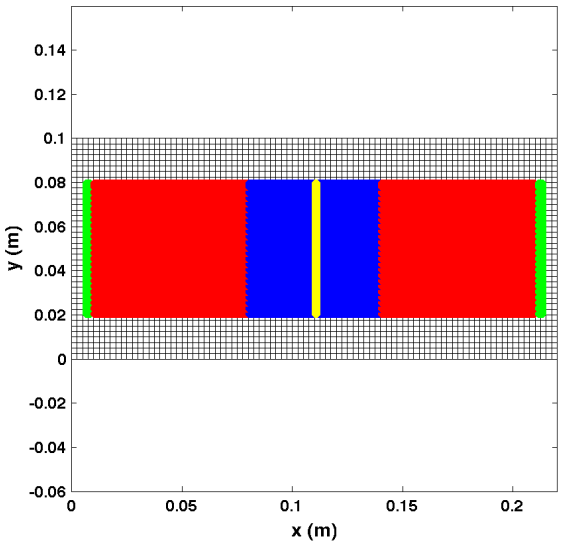


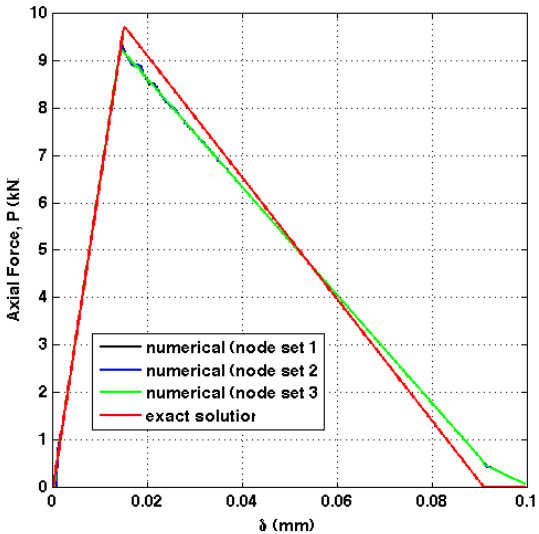
$$E_P = \frac{(P_{mpm} - P_{analytical})}{P_{analytical}}$$ axial load error

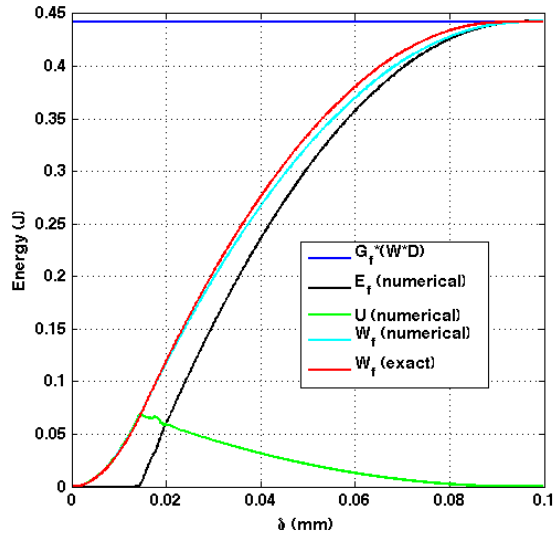
h square grid cell side length

Grid Orientation Bias: Stiffening Effect

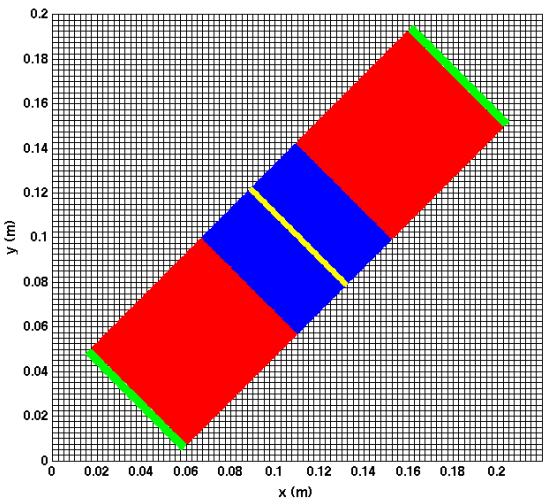
$\alpha = 0°$

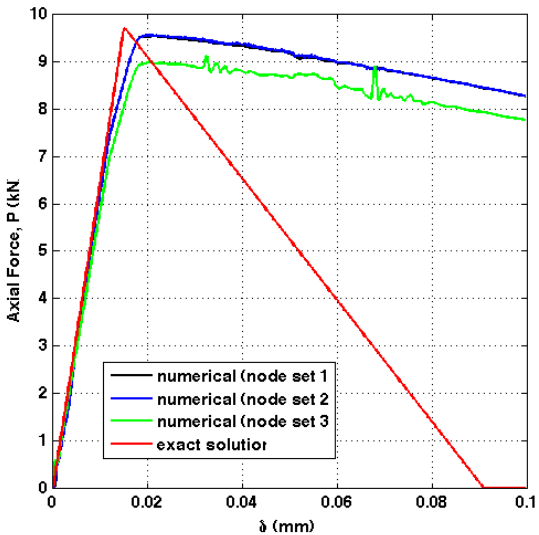


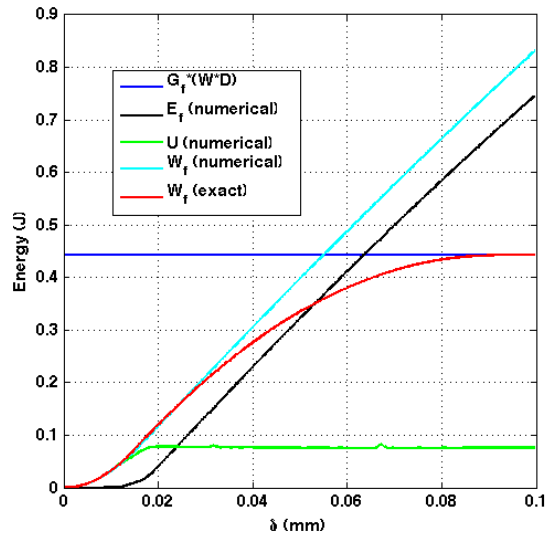




$\alpha = 45°$

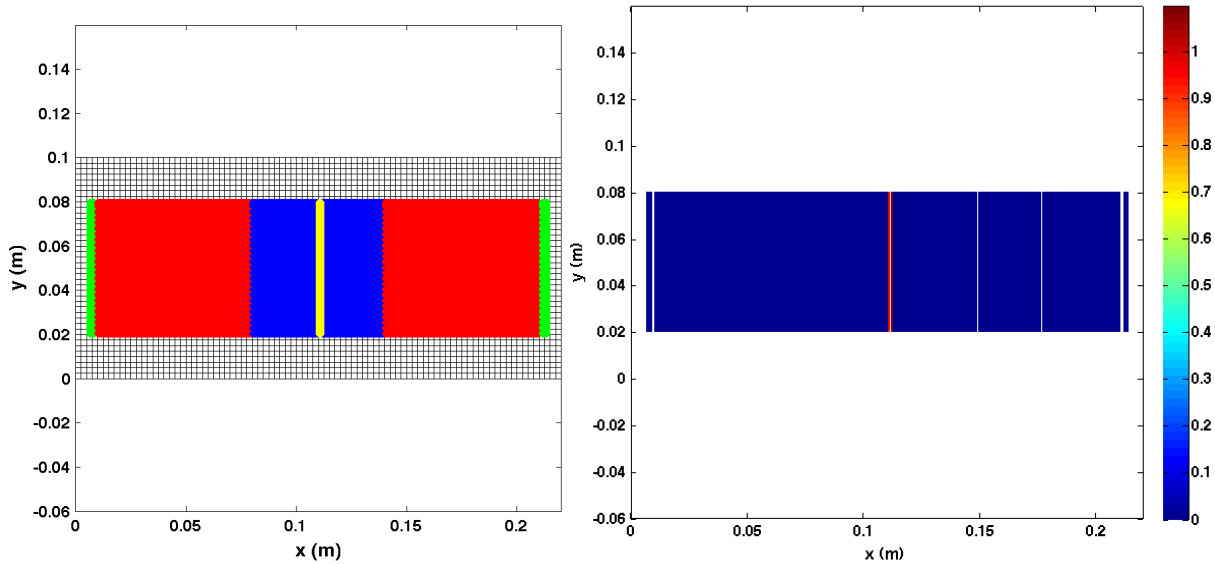




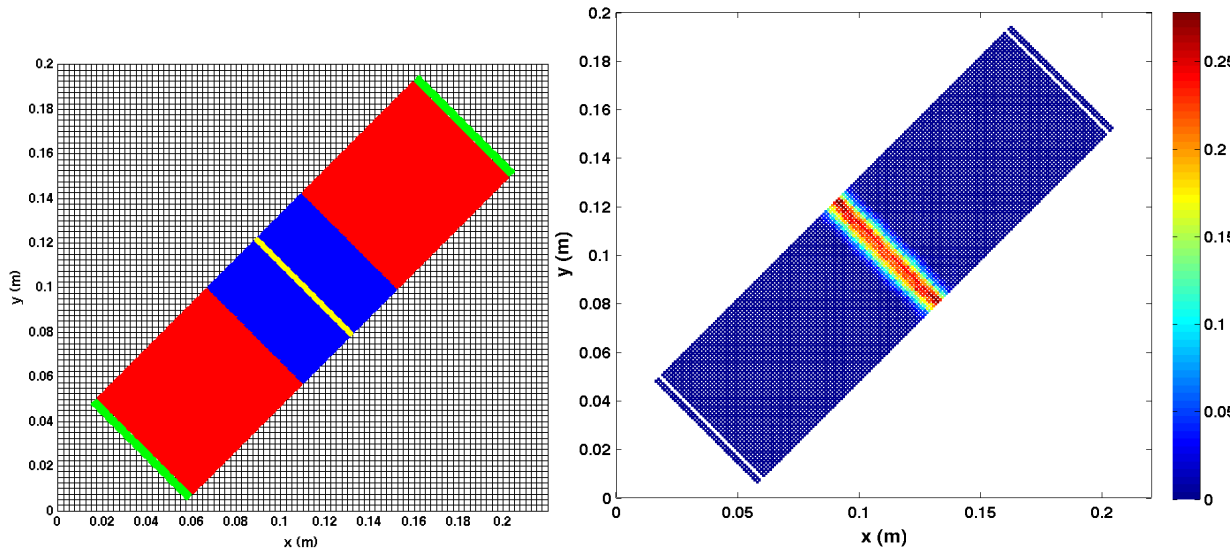


Grid Orientation Bias: Failure Pattern Spreading

$[[u_n]]/u_0$ for $\alpha = 0°$

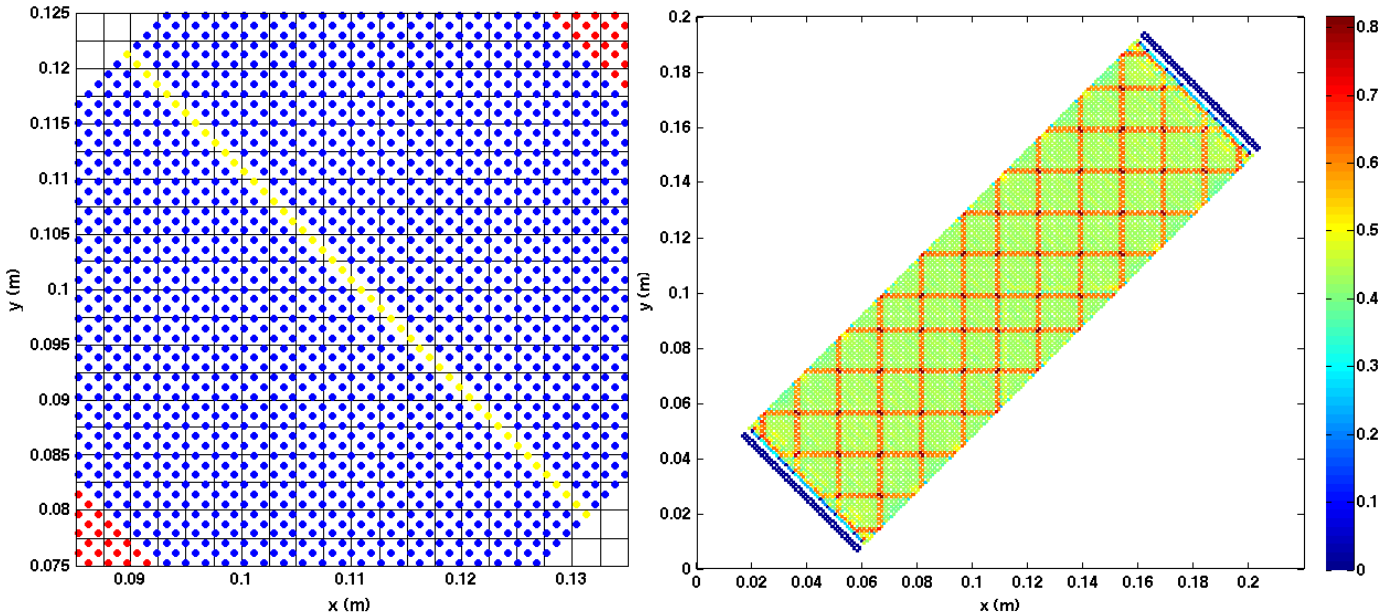


$[[u_n]]/u_0$ for $\alpha = 45°$

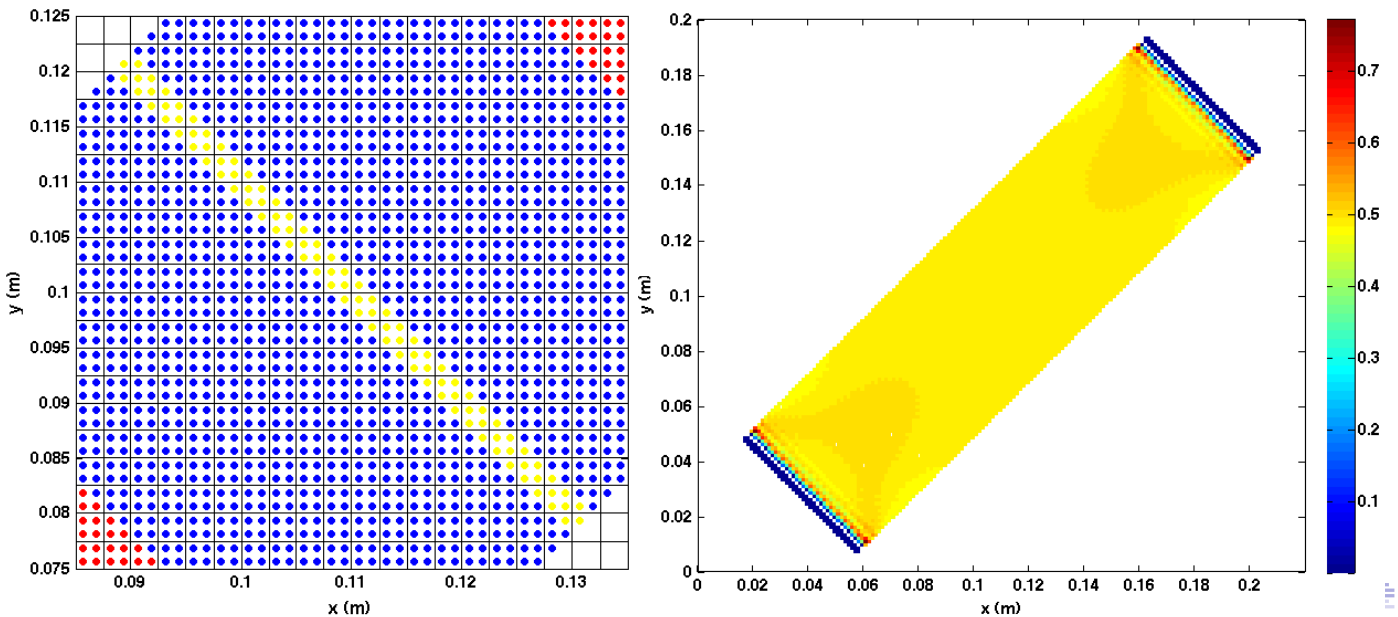


In-cell Material Point Distribution Effect on Stress Field

non-uniform in-cell material point distribution and σ_{nn}/τ_{nf}



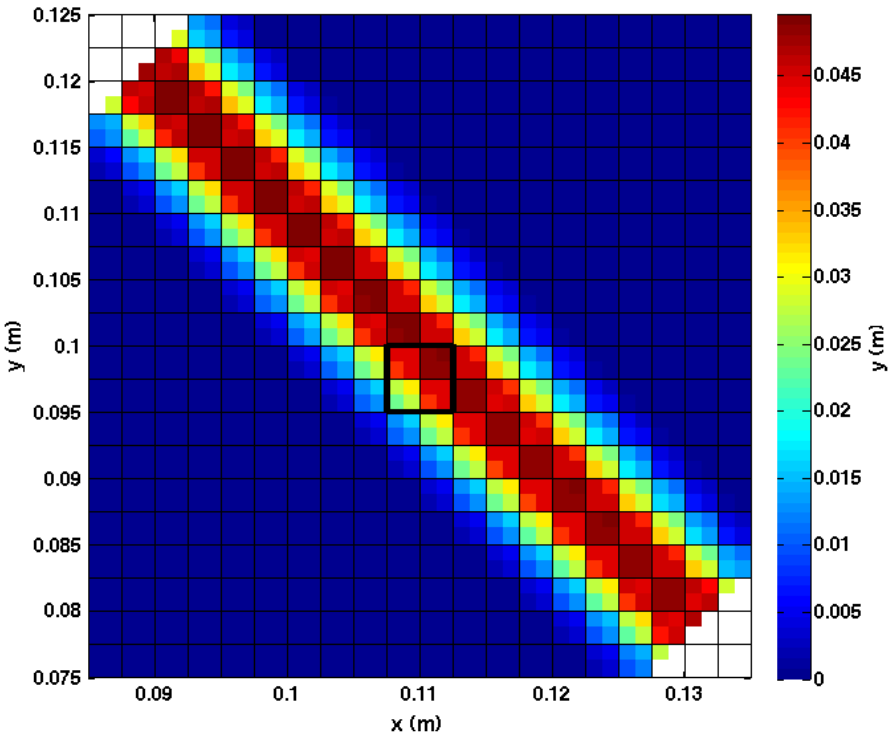
uniform in-cell material point distribution and σ_{nn}/τ_{nf}



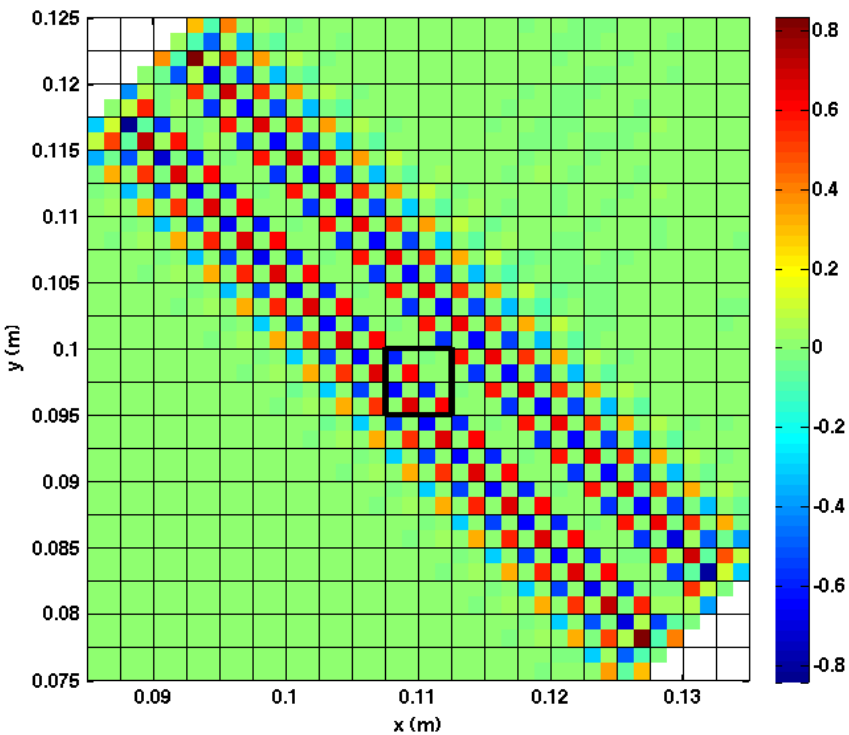
Investigation of failure zone

- Unphysical build-up of stress in failure region and spreading of failure zone

$[[u_n]]/u_0$

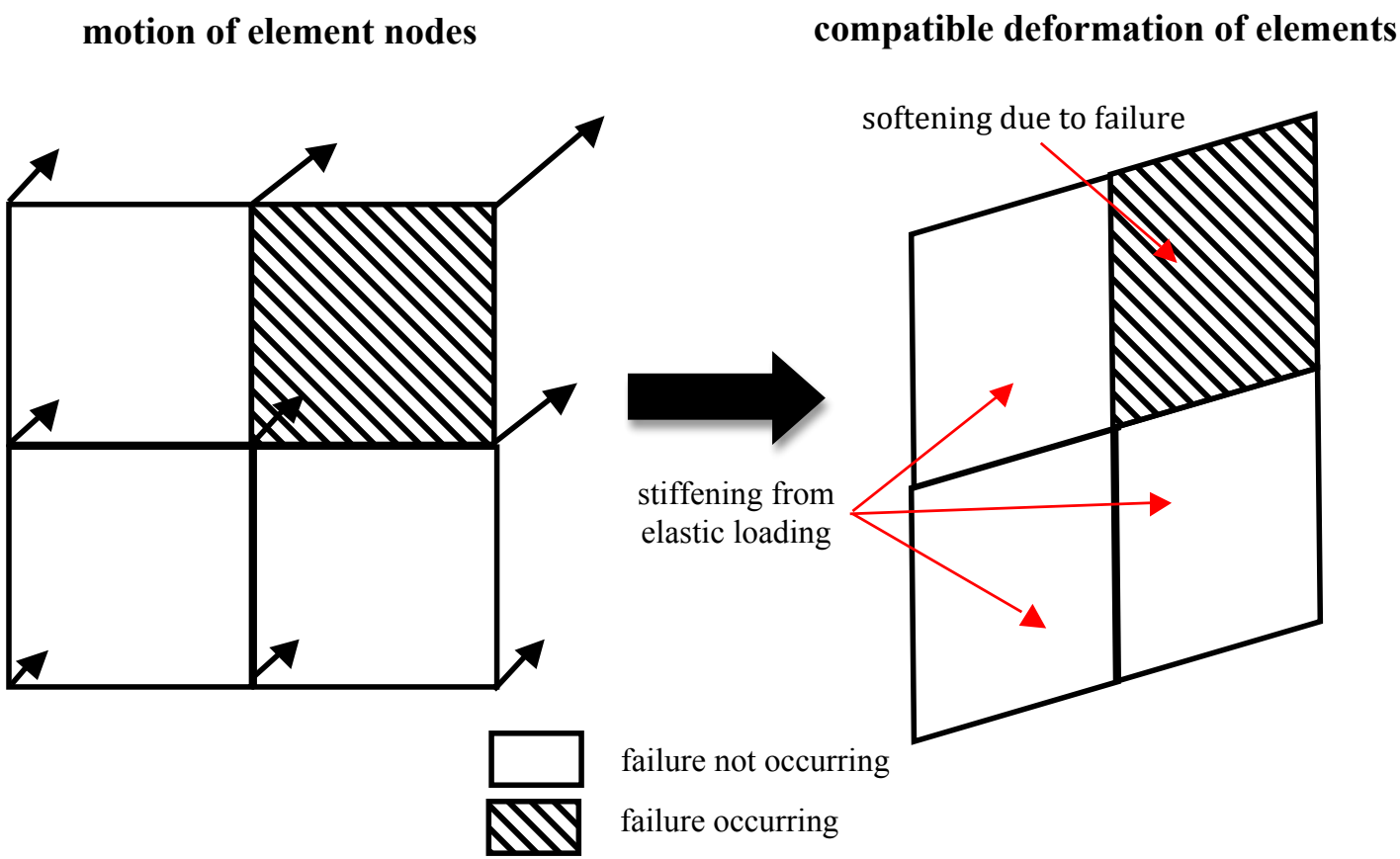


σ_{nn}/τ_{nf}



Compatibility

- Stress build-up $\rightarrow$ stiffening
- Stress build-up in smeared crack MPM caused by compatibility of deformation imposed by the grid
- Stress build-up $\longleftrightarrow$ spreading of failure zone
- "Stress locking" in smeared crack FEM [Rots, 1988]



Conclusions & Future Work

- Smeared crack MPM suffers from grid orientation bias
  - A stiffening and spreading of failure pattern occurs when failure surface and grid cell lines are not aligned
- Limited use of smeared crack MPM is reccomended
- Embedded discontinuity [Oliver, 1996], [Oliver et al., 2003] MPM unsuccessful
- Future approaches for representing failure in MPM

Bazant, Z. and Oh, B. (1983).

Crack band theory for fracture of concrete.
RILEM Materials and Structures, 16:155–177.

de Borst, R. and Nauta, P. (1985).

Non-orthogonal cracks in a smeared finite element model.
Engineering Computations, 2:35–46.

Hillerborg, A., Modeer, M., and Petersson, P. (1976).

Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements.
Cement and Concrete Research, 6:773–782.

Oliver, J. (1996).

Modeling strong discontinuities in solid mechanics via strain softening constitutive equations. part 2: numerical simulation.
International Journal for Numerical methods in engineering, 39:3601–3623.

Oliver, J., Huespe, A., and Samaniego, E. (2003).

A study on finite elements for capturing strong discontinuities.
International Journal for Numerical methods in engineering, 56:2135–2161.

Peterson, K. (2008).

Modeling arctic sea ice using the material-point method and a elastic-decohesive rheology.
PhD thesis, University of New Mexico, Albuquerque, New Mexico, U.S.A.

Rots, J. (1988).

Smeared Computational modeling of concrete fracture.
PhD thesis, Delft University of Technology, Delft, The Netherlands.

Schreyer, H. (2007).

Modeling surface orientation and stress at failure of concrete and geological materials.
Journal For Numerical And Analytical Methods In Geomechanics, 31:144–171.

Sulsky, D. and Kaul, A. (2004).

Implicit dynamics in the material-point method.
Comp. Meths. Appld. Mechs. Engrg., 139:1137–1170.

Sulsky, D., Schreyer, H., Peterson, K., Coon, M., and Kwok, R. (2007).

Using the material-point method to model sea ice dynamics.
J. Geophys. Res., 112.