Disease Control & Eradication

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One of the main reasons for studying infectious disease is to improve control and potentially eradicate the infection. Mathematical models can help us to understand the impacts of different control measures and to develop better control strategies. All types of control are aimed at reducing the transmission of the pathogen between susceptible and infectious individuals.

Let's list the major types of control:

- Vaccination - reduces number of susceptible individuals
- Quarantine or isolation - reduces number of suspected exposed or infected individuals mixing in the general population
- Social distancing - reduces transmission rate between susceptible and infectious individuals
- Anti-microbials - can reduce infectiousness/duration of infection in infectious individuals and may be given as prophylaxis to susceptibles
- Culling (plant and non-human animals only!) - reduces number of susceptible and infected individuals

Contract tracing (tracing the contacts of infective individuals) may be used together with any of these measures.

Vaccination

First, let's consider how we might include vaccination in the SIR model with demography. We'll assume that we vaccinate a proportion p of newborns. The equations are given by

$$\frac{dS}{dt} = \nu(1-p)N - \beta\frac{I}{N}S - \mu S \quad (1)$$
$$\frac{dI}{dt} = \beta\frac{I}{N}S - (\gamma+\mu)I \quad (2)$$
$$\frac{dR}{dt} = \nu pN + \gamma I - \mu R\ . \quad (3)$$

We're going to use a simple change of variables to show how vaccination is dynamically equivalent to reducing the population's birth rate or the transmission rate. The change of variables that we'll use is the following:

$$S(t) = \hat{S}(t)(1-p), \quad I(t) = \hat{I}(1-p), \quad R = \hat{R}(1-p) + \frac{\nu}{\mu}p\ .$$

Let's the make the substitutions into the equations above:

$$\begin{aligned}
\frac{d\hat{S}}{dt}(1-p) &= \nu(1-p)N - \beta\frac{\hat{I}}{N}(1-p)\hat{S}(1-p) - \mu\hat{S}(1-p) \\
\frac{d\hat{I}}{dt}(1-p) &= \beta\frac{\hat{I}}{N}(1-p)\hat{S}(1-p) - (\gamma+\mu)\hat{I}(1-p) \\
\frac{d\hat{R}}{dt}(1-p) &= \nu p + \gamma\hat{I}(1-p) - \mu\hat{R}(1-p) - \nu p\ .
\end{aligned}$$

Simplifying, by dividing through by $(1-p)$:

$$\begin{aligned}
\frac{d\hat{S}}{dt} &= \nu N - \beta(1-p)\frac{\hat{I}}{N}\hat{S} - \mu\hat{S} \\
\frac{d\hat{I}}{dt} &= \beta(1-p)\frac{\hat{I}}{N}\hat{S} - (\gamma+\mu)\hat{I} \\
\frac{d\hat{R}}{dt} &= \gamma\hat{I} - \mu\hat{R}\ .
\end{aligned}$$

Leads to $R_0' = (1-p)R_0$. Critical vaccination coverage is defined by $1 = (1-p_c)R_0$. So $p_c = 1 - 1/R_0$.

Important: to eradicate an infection, it is not necessary to vaccinate the entire population.

Relation to average age at infection, and perverse outcomes of vaccination (rubella) when effects of disease are age-dependent.

****Try This****

1. Choose one of the other types of control and modify the SIR model to include its effects.