

311-Homework 27 answers

Name: _____

May 2, 2005

1. **Problem 5.2.1, p. 294**

Since $\Delta\phi = 0$ and the region contains the origin, the integral equals (by Greens 3rd identity):

$$I = \phi(0,0,0) = 5 .$$

2. **Problem 5.2.2, p.284**

As in the previous problem, both potentials are harmonic and both regions include the origin; so by Greens 3rd identity:

(a) $I = \phi(0,0,0) = 4$

(b) $I = \phi(0,0,0) = 5.$

3. **Problem 5.2.8a, p.285**

We have:

$$\nabla \cdot (\phi \nabla \times \mathbf{E}) = \nabla \phi \cdot \nabla \times \mathbf{E}$$

since $\nabla \cdot \nabla \times \mathbf{E} = 0$, so that

$$\int \int \int_D \nabla \phi \cdot \nabla \times \mathbf{E} dV = \int \int \int_D \nabla \cdot (\phi \nabla \times \mathbf{E}) dV = \int \int_S \phi \nabla \times \mathbf{E} \cdot d\mathbf{S}$$

while

$$\nabla \times (\phi \mathbf{E}) = \phi \nabla \times \mathbf{E} + \nabla \phi \times \mathbf{E}$$

so that

$$\int \int_S \nabla \times (\phi \mathbf{E}) \cdot d\mathbf{S} = \int \int_S \nabla \phi \times \mathbf{E} \cdot d\mathbf{S} + \int \int_S \phi \nabla \times \mathbf{E} \cdot d\mathbf{S}$$

The integral on the left vanishes since

$$\int \int_S \nabla \times (\phi \mathbf{E}) \cdot d\mathbf{S} = \int \int \int_D \nabla \cdot (\nabla \times (\phi \mathbf{E})) dV = 0$$

which leaves us with

$$\int \int \int_D \nabla \phi \cdot \nabla \times \mathbf{E} dV = \int \int_S \phi \nabla \times \mathbf{E} \cdot d\mathbf{S} = - \int \int_S \nabla \phi \times \mathbf{E} \cdot d\mathbf{S} = \int \int_S \mathbf{E} \times \nabla \phi \cdot d\mathbf{S}$$

4. **Problem S6.74**, p.134

direct application of divergence theorem, since

$$\nabla \cdot \mathbf{E} = - \Delta \phi$$

Now use lemma 5.4, p. 281 in Davis and Snider.