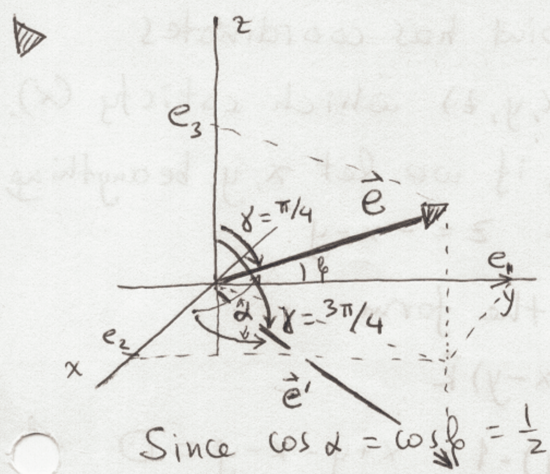
Math. 311
Spring 99

Homework 1
1.5(4,16), 1.7(1,5,9)

p.14, 1.5.4 $\vec{B} = 2\vec{i} + 2\vec{j} - \vec{k}$
For what values of s is $|sB| = 1$?

$|s\vec{B}| = |s||\vec{B}| = |s|\sqrt{2^2+2^2+1^2} = |s|\cdot 3 = 1 \Rightarrow s = \pm\frac{1}{3}$

p.14, 1.5.16 How many unit vectors are there for which $\cos\alpha = \frac{1}{2}$ and also $\cos\beta = \frac{1}{2}$? Illustrate with a diagram.



Let $\vec{e}$ unit; $\vec{e} = e_1\vec{i} + e_2\vec{j} + e_3\vec{k}$

Then $|\vec{e}|^2 = e_1^2 + e_2^2 + e_3^2 = 1$.

Since $\frac{e_1}{|\vec{e}|} = \cos\alpha$, $\frac{e_2}{|\vec{e}|} = \cos\beta$, $\frac{e_3}{|\vec{e}|} = \cos\gamma$

we have the law of cosines:

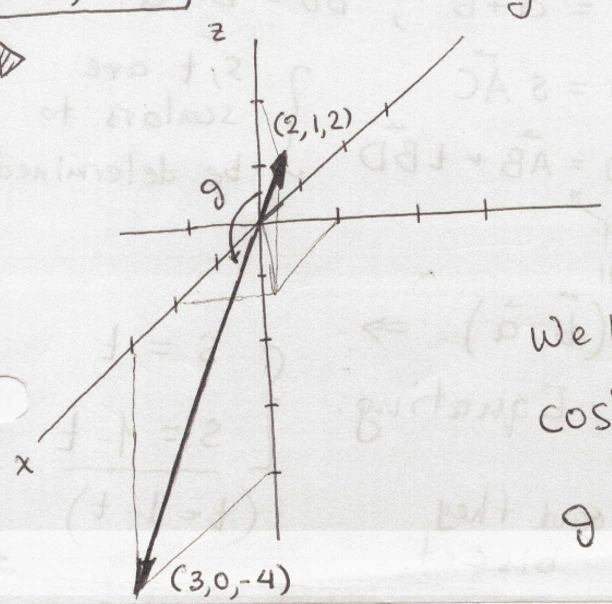
$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.}$$

Since $\cos\alpha = \cos\beta = \frac{1}{2}$, we have $\cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$

$\Rightarrow \cos\gamma = \pm\frac{1}{\sqrt{2}} \Rightarrow \gamma = \begin{cases} \pi/4 \\ 3\pi/4 \end{cases}$

$\therefore$ two vectors: $\vec{e} = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$; $\vec{e}' = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$

p.23, 1.7.1 Find the angle between $2\vec{i} + \vec{j} + 2\vec{k}$ and $3\vec{i} - 4\vec{k}$.



Let $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

$\vec{b} = 3\vec{i} - 4\vec{k}$

Then $|a| = \sqrt{2^2+1^2+2^2} = 3$

$|b| = \sqrt{3^2+0^2+4^2} = 5$

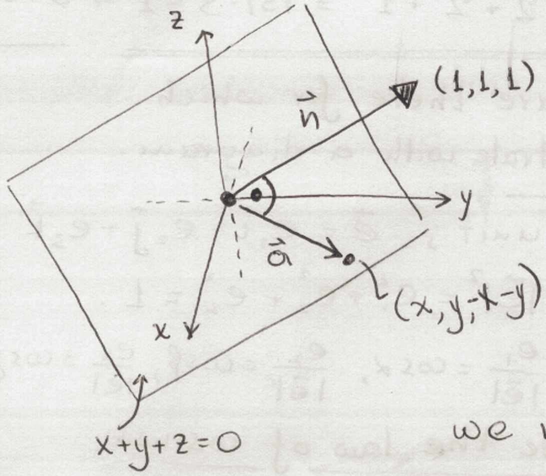
We have $\vec{a}\cdot\vec{b} = 2\cdot 3 + 1\cdot 0 + 2\cdot(-4) = -2$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-2}{3\cdot 5} = -\frac{2}{15}$

$\theta = \cos^{-1}\left(-\frac{2}{15}\right) =$

(2/2)

p.23, 1.7.5 Show that $\vec{i}+\vec{j}+\vec{k}$ is perpendicular to the plane $x+y+z=0$.



Let $\vec{n} = \vec{i}+\vec{j}+\vec{k}$

▶ Consider an arbitrary point on the plane

$$x+y+z=0. \quad (*)$$

This means that this point has coordinates (x, y, z) which satisfy $(*)$.

Then, if we let x, y be anything we must have $z = -x-y$.

So, any vector on the plane has the form

$$\vec{a} = x\vec{i} + y\vec{j} + (-x-y)\vec{k}$$

and $\vec{a} \bullet \vec{n} = x \cdot 1 + y \cdot 1 + (-x-y) \cdot 1 = x+y-x-y = 0$ ◀

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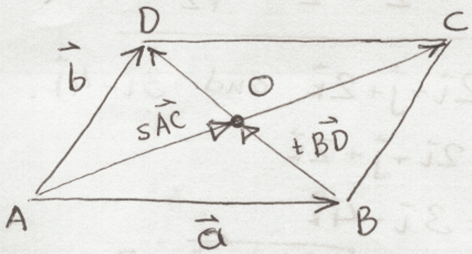
p.24, 1.7.9 ◀ Let $\left.\begin{array}{l}\vec{AB} = \vec{DC} = \vec{a} \\ \vec{AD} = \vec{BC} = \vec{b}\end{array}\right\}$ not collinear



"Show that the diagonals of a parallelogram bisect each other"

The diagonals are

$$\vec{AC} = \vec{a}+\vec{b} \; ; \; \vec{BD} = \vec{b}-\vec{a}$$

$$\left.\begin{array}{l}\text{Let } \vec{AO} = s\,\vec{AC} \\ \text{or } \vec{AO} = \vec{AB} + t\vec{BD}\end{array}\right\} s, t \text{ are scalars to be determined.}$$

Setting these equal:

$$s\vec{AC} = s(\vec{a}+\vec{b}) = \vec{a} + t(\vec{b}-\vec{a}) \Rightarrow$$

$$\Rightarrow s\vec{a} + s\vec{b} = \underbrace{(1-t)\vec{a} + t\vec{b}}. \text{ Equating: } \left\{\begin{array}{l} s = t \\ s = 1-t \end{array}\right. \quad (t = 1-t)$$

$$\Rightarrow \underbrace{s = t = \frac{1}{2}} \text{ and they bisect.}$$ ◀