SUPPLEMENTARY PROBLEMS

37. Verify Green's theorem in the plane for $\oint_C (3x^2-8y^2)\,dx + (4y-6xy)\,dy$, where C is the boundary of the region defined by: (a) $y=\sqrt{x}$, $y=x^2$; (b) $x=0$, $y=0$, $x+y=1$.
Ans. (a) common value = 3/2 (b) common value = 5/3

38. Evaluate $\oint_C (3x+4y)\,dx + (2x-3y)\,dy$ where C, a circle of radius two with center at the origin of the xy plane, is traversed in the positive sense. *Ans.* -8π

39. Work the previous problem for the line integral $\oint_C (x^2+y^2)\,dx + 3xy^2\,dy$. *Ans.* 12π

40. Evaluate $\oint (x^2-2xy)\,dx + (x^2y+3)\,dy$ around the boundary of the region defined by $y^2=8x$ and $x=2$ (a) directly, (b) by using Green's theorem. *Ans.* 128/5

41. Evaluate $\int_{(0,0)}^{(\pi,2)} (6xy-y^2)\,dx + (3x^2-2xy)\,dy$ along the cycloid $x=\theta-\sin\theta$, $y=1-\cos\theta$.
Ans. $6\pi^2-4\pi$

42. Evaluate $\oint (3x^2+2y)\,dx - (x+3\cos y)\,dy$ around the parallelogram having vertices at (0,0), (2,0), (3,1) and (1,1). *Ans.* -6

43. Find the area bounded by one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, $a>0$, and the x axis.
Ans. $3\pi a^2$

44. Find the area bounded by the hypocycloid $x^{2/3}+y^{2/3}=a^{2/3}$, $a>0$.
Hint: Parametric equations are $x=a\cos^3\theta$, $y=a\sin^3\theta$. *Ans.* $3\pi a^2/8$

45. Show that in polar coordinates (ρ,ϕ) the expression $x\,dy-y\,dx=\rho^2\,d\phi$. Interpret $\frac{1}{2}\int x\,dy-y\,dx$.

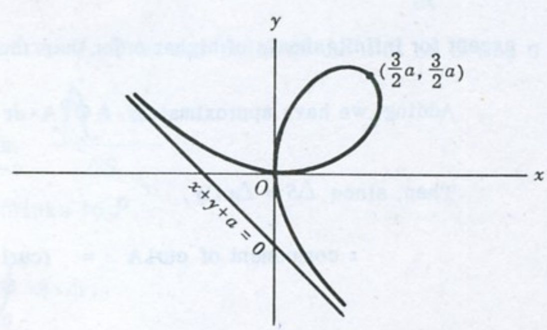
46. Find the area of a loop of the four-leafed rose $\rho=3\sin 2\phi$. *Ans.* $9\pi/8$

47. Find the area of both loops of the lemniscate $\rho^2=a^2\cos 2\phi$. *Ans.* a^2

48. Find the area of the loop of the folium of Descartes $x^3+y^3=3axy$, $a>0$ (see adjoining figure).
Hint: Let $y=tx$ and obtain the parametric equations of the curve. Then use the fact that

$$\text{Area} = \tfrac{1}{2}\oint x\,dy - y\,dx = \tfrac{1}{2}\oint x^2\,d\left(\frac{y}{x}\right) = \tfrac{1}{2}\oint x^2\,dt$$

Ans. $3a^2/2$



49. Verify Green's theorem in the plane for $\oint_C (2x-y^3)\,dx - xy\,dy$, where C is the boundary of the region enclosed by the circles $x^2+y^2=1$ and $x^2+y^2=9$. *Ans.* common value = 60π

50. Evaluate $\int_{(1,0)}^{(-1,0)} \frac{-y\,dx + x\,dy}{x^2+y^2}$ along the following paths: