1. Let S_t be a uniformly expanding hemisphere described by

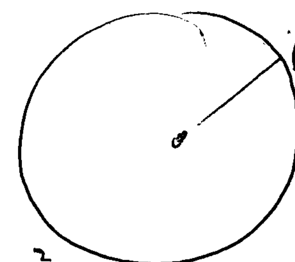
$$x^2+y^2+z^2=(vt)^2 \quad , \; z \geq 0$$

And let $\vec{F}$ be the vector field:

$$F(R,t) = \vec{R}\,t$$

Verify the flux transport thm. in this case.

here $\vec{V}\Big|_{R,t} = v\vec{R}/R$; $d\vec{S} = \frac{\vec{R}}{R}\,dS$



$\nabla\cdot F = 3t$; $F\times\vec{V} = 0$

$$\phi_t = \iint_{S_t} F\cdot d\vec{S} = \iint_{S_t} \underset{\underset{vt^2\cancel{\frac{\vec{R}}{R}}\cdot\frac{\vec{R}}{R}ds}{\parallel}}{R(t)ds} = vt^2\cdot 4\pi\underset{\underset{R}{\parallel}}{(vt)}^2$$

$$= 4\pi(v^3t^4)$$

$$\vec{F} = \vec{R}t$$

$$\vec{R} = vt\frac{\vec{R}}{R}$$

$$\frac{d\phi}{dt} = 16\pi v^3t^3 = 2\pi(2vt)^3$$

$$\vec{F} = \vec{R}t$$

$$\frac{\partial F}{\partial t} = \vec{R} \;;\; \iint_{S_t} \underset{\underset{\vec{R}=vt\cancel{\frac{\vec{R}}{R}}\cdot\frac{\vec{R}}{R}ds}{\parallel}}{\frac{\partial F}{\partial t}\cdot dS} = vt\cdot 4\pi(vt)^2 = 4\pi(vt)^3$$

$$\iint \nabla\cdot\vec{F}\, v\cdot dS = \iint_{S_t} 3t\,v\,dS = 3vt\cdot 4\pi(vt)^2$$

$$\boxed{\frac{d\phi}{dt} = \iint_{S_t}\left(\frac{\partial F}{\partial t} + (\nabla\cdot F)v + \nabla\times(F\times v)\right)dv}$$