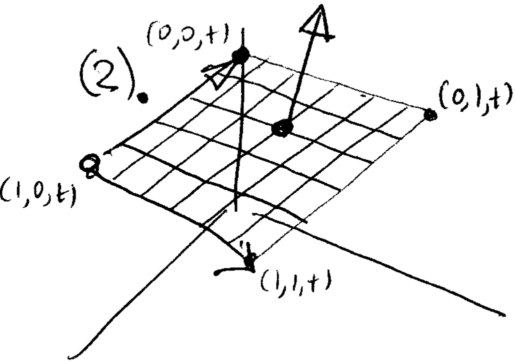
(2).



$\vec{F} = xz\,\vec{k}$

$\vec{N} = [(1,1,t)-(1,0,t)]\times[(0,0,t)-(1,0,t)]$

$(0,1,0)\times(-1,0,0) = \vec{k}$

square

$d\vec{S} = \dfrac{\vec{N}\,dx\,dy}{\vec{N}\cdot\vec{k}} = \vec{k}\,dx\,dy$

$x(t)=x,\ z(t)=t \qquad \vec{F} = xt\,\vec{k}$

$\dfrac{\partial F}{\partial t} = 0$

$\phi = \iint_{S_t} F\cdot\vec{k}\,dS = \iint xt\,dx\,dy$

$= t\,\frac{x^2}{2}\Big|_0^1 = \dfrac{t}{2}$

$\dfrac{d\phi}{dt} = \dfrac{1}{2}\ ; \quad \dot{\nabla}\cdot F = x\ , \quad \vec{v} = \vec{k}$

$\iint \nabla\cdot F\ \vec{v}\cdot\vec{k}\,dS = \iint x\,dx\,dy = \dfrac{1}{2}$

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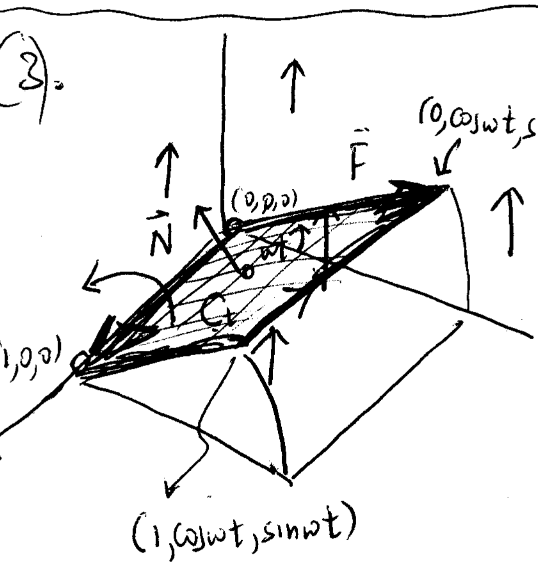
(3).



Square: $\vec{i}\times(\cos\omega t\,\vec{j} + \sin\omega t\,\vec{k}) = \vec{n}(t$

$\vec{n} = -\sin\omega t\,\vec{j} + \cos\omega t\,\vec{k} \qquad dS = \cancel{\cos\omega}$

$\phi = \iint_{S_t} \vec{F}\cdot\vec{n}\,dS = \iint \dfrac{n\cdot k}{n\cdot k}\,dx\,dy = \iint dx\,dy$

$= \int_{x=0}^{1} dx \int_{y=0}^{\cos\omega t} dy = \cos\omega t\ ; \quad \dfrac{d\phi}{dt} = -\omega\sin\omega t$

$\iint \quad \cancel{\vec{F}\times\vec{v} =} \quad \vec{v} = \omega\,\vec{i}\times\vec{R} = \omega(-z\vec{j} + y\vec{k})$

$\vec{F}\times\vec{v} = \omega\,\vec{k}\times(\underbrace{\vec{i}\times\vec{R}}_{-z\vec{j}+y\vec{k}}) = \omega z\,\vec{i}$

$\nabla\times(F\times\vec{v}) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \omega z & & \end{vmatrix} = \omega\vec{j}$

$\iint \nabla\times(F\times v)\cdot d\vec{S} = \omega\int_{x=0}^{1}\int_{y=0}^{\cos\omega t} \dfrac{\vec{j}\cdot\vec{n}}{k\cdot n}\,dx\,dy = \omega\int_{x=0}^{1} \dfrac{-\sin\omega t}{\cos\omega t}\,\cancel{\cos\omega t}\,dt = -\omega\sin\omega$