311-Homework 27 answers

Name:____

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1. **Problem 5.2.1, p. 294**
   Since $\triangle\phi = 0$ and the region contains the origin, the integral equals (by Greens 3rd identity):
   $$I = \phi(0,0,0) = 5 \ .$$

2. **Problem 5.2.2, p.284**
   As in the previous problem, both potentials are harmonic and both regions include the origin; so by Greens 3rd identity:
   (a) $I = -4\pi\phi(0,0,0) = -16\pi$
   (b) $I = -4\pi\phi(0,0,0) = -20\pi$.

3. **Problem 5.2.8a, p.285**
   We have:
   $$\nabla \cdot (\phi \nabla \times \mathbf{E}) = \nabla\phi \cdot \nabla \times \mathbf{E}$$
   since $\nabla \cdot \nabla \times \mathbf{E} = 0$, so that
   $$\int\int\int_D \nabla\phi \cdot \nabla \times \mathbf{E} dV = \int\int\int_D \nabla \cdot (\phi\nabla \times \mathbf{E}) dV = \int\int_S \phi\nabla \times \mathbf{E} \cdot d\mathbf{S}$$
   while
   $$\nabla \times (\phi\mathbf{E}) = \phi\nabla \times \mathbf{E} + \nabla\phi \times \mathbf{E}$$
   so that
   $$\int\int_S \nabla \times (\phi\mathbf{E}) \cdot d\mathbf{S} = \int\int_S \nabla\phi \times \mathbf{E} \cdot d\mathbf{S} + \int\int_S \phi\nabla \times \mathbf{E} \cdot d\mathbf{S}$$
   The integral on the left vanishes since
   $$\int\int_S \nabla \times (\phi\mathbf{E}) \cdot d\mathbf{S} = \int\int\int_D \nabla \cdot (\nabla \times (\phi\mathbf{E})) dV = 0$$

which leaves us with

$$\int\int\int_D \nabla\phi\cdot\nabla\times\mathbf{E}dV = \int\int_S \phi\nabla\times\mathbf{E}\cdot d\mathbf{S} = -\int\int_S \nabla\phi\times\mathbf{E}\cdot d\mathbf{S} = \int\int_S \mathbf{E}\times\nabla\phi\cdot d\mathbf{S}$$

4. **Problem S6.74, p.134**
   direct application of divergence theorem, since

$$\nabla\cdot\mathbf{E} = -\triangle\,\phi$$

   Now use lemma 5.4, p. 281 in Davis and Snider.