Solutions, 311-XXVIII

December 2, 2011

Sec. 5.(5) p.299(1*,2*,3*), Schaum's p.134(63*,64,65*,66)

1 Problem 5.5.3

Prove
$$\int\int_S \nabla\phi \times \nabla\psi \cdot d\mathbf{S} = \oint_C \phi\nabla\psi \cdot d\mathbf{R}\ .$$

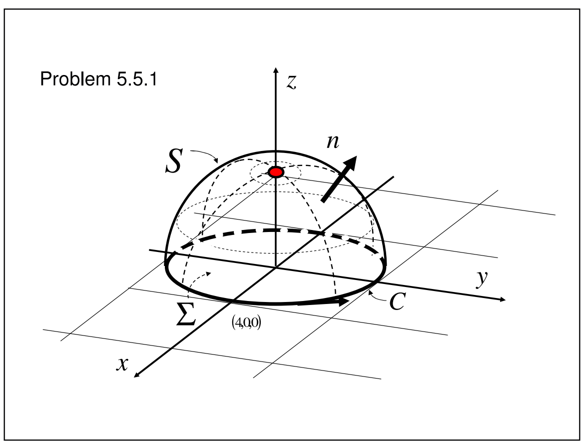
Solution:

2 Problem 5.5.1

Given the vector field $\mathbf{F} = 3y\mathbf{i} + (5 - 2x)\mathbf{j} + (z^2 - 2)\mathbf{k}$, find

1. $\nabla \cdot \mathbf{F}$.
2. $\nabla \times \mathbf{F}$.
3. The surface integral of the normal component of $\nabla \times \mathbf{F}$ over the open hemispherical surface $x^2 + y^2 + z^2 = 4$ above the xy plane.

[*Hint:* By a double application of Stokes' theorem, part (3) can be reduced to a triviality.]
Solution:

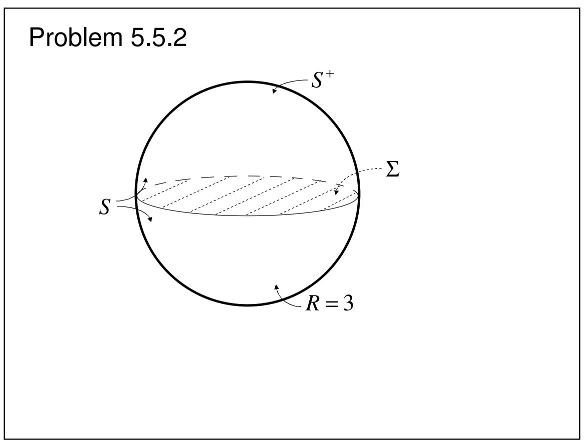


3 Problem 5.5.2

Given that $\nabla \times \mathbf{F} = 2y\mathbf{i} - 2x\mathbf{j} + 3\mathbf{k}$, find the surface iintegral of the normal component of $\nabla \times \mathbf{F}$ (not $\mathbf{F}$!) over

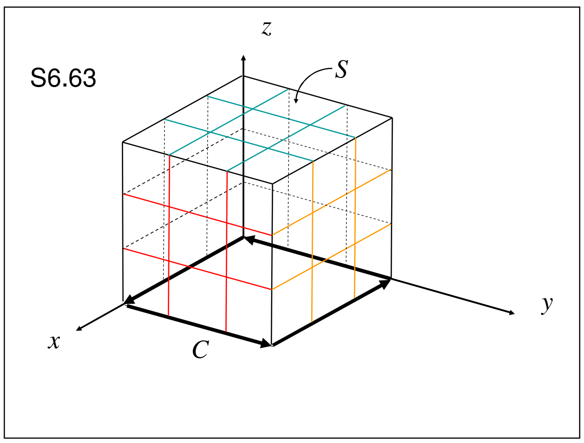
1. The open hemispherical surface $x^2 + y^2 + z^2 = 9$, $z > 0$.
2. The sphere $x^2 + y^2 + z^2 = 9$.

Solution:



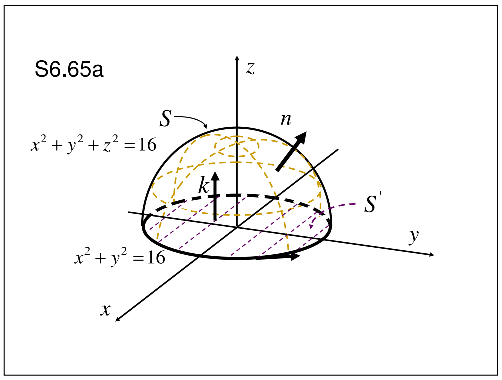
4 Problem S6.63, p.134

Verify Stokes' theorem for $\mathbf{A} = (y - z + 2)\mathbf{i} + (yz + 4)\mathbf{j} - xz\mathbf{k}$ where S is the surface of the cube $x = 2$, $y = 2$, $z = 2$, $x = 0$, $= 0$, $z = 0$, over the xy plane (i.e. the face $z = 0$ is open).
Solution:



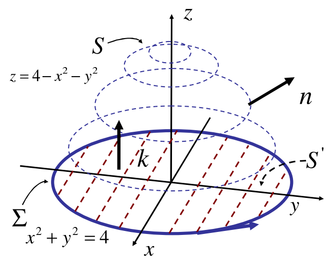
5 Problem S6.65, p.134

Evaluate $\int\int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n} dS$ where $\mathbf{A} = (y - z + 2)\mathbf{i} + (yz + 4)\mathbf{j} - xz\mathbf{k}$ and S is the surface of (a) the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane and (b) the paraboloid $z = 4 - (x^2 + y^2)$ above the xy plane.
Solution:



1

S6.65b



1