311 Fall 2006 – TEST II

Name: ____

November 1, 2006

INSTRUCTIONS:
ALL PROBLEMS ARE WEIGHTED EQUALLY!
DO ALL FOUR PROBLEMS!
One full page of notes is allowed.

Problem	grade
1	
2	
3	
4	
5	
Total	

(**1 − 25** pts) Compute scalar and vector potentials for the 2-dimensional field

$$\mathbf{F} = e^x \sin y \mathbf{i} + e^x \cos y \mathbf{j}$$

(2 − 25 pts) Evaluate the integral

$$\int\int_S ((y+1)\mathbf{i} + (2x+1)\mathbf{j} + z\mathbf{k}) \cdot d\mathbf{S}$$

where S is the surface of the triangle with vertices at $(2,0,0)$, $(0,0,2)$, $(0,1,0)$ and normal pointing away from the origin.

(**3 − 25** pts) Compute the flux of the vector field

$$\mathbf{F} = ze^{y}\mathbf{i} - xz\cos z\mathbf{j} + (z+1)\mathbf{k}$$

over the open hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.
(hint: use the divergence theorem to relate the desired integral to the sum of an easier surface integral and a volume integral).

($4 - \mathbf{25}$ pts) Find $I := \oint_C \mathbf{F} \cdot d\mathbf{R}$ where C is the ellipse of intersection of the cylinder $x^2+y^2 = 1$ with the plane $z = y$ and $\mathbf{F} = x\mathbf{i}+(x+y)\mathbf{j}+(x+y+z)\mathbf{k}$:
(a $-$ **12** pts) By direct evaluation of the line integral.

(b $-$ **13** pts) By applying Stoke's theorem to convert to a surface integral over an appropriate surface.