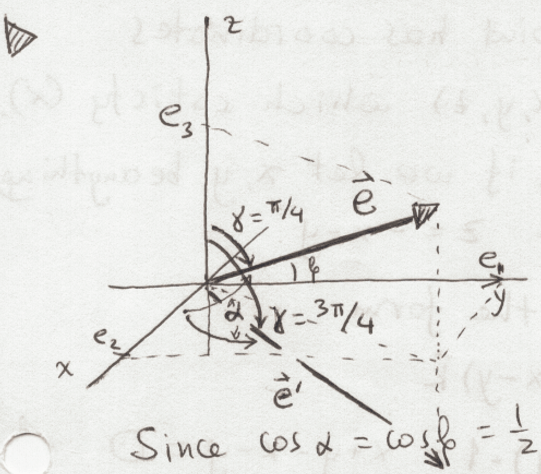
Math. 311
Spring 99
Homework 1
1.5(4,16), 1.7(1,5,9)

p.14, 1.5.4 $\vec{B} = 2\vec{i} + 2\vec{j} - \vec{k}$

For what values of s is $|s\vec{B}| = 1$?

$$|s\vec{B}| = |s||\vec{B}| = |s|\sqrt{2^2+2^2+1^2} = |s|\cdot 3 = 1 \Rightarrow s = \pm\frac{1}{3}$$

p.14, 1.5.16 How many unit vectors are there for which $\cos\alpha = \frac{1}{2}$ and also $\cos\beta = \frac{1}{2}$? Illustrate with a diagram.



Let $\vec{e}$ unit; $\vec{e} = e_1\vec{i} + e_2\vec{j} + e_3\vec{k}$

Then $|\vec{e}|^2 = e_1^2 + e_2^2 + e_3^2 = 1$.

Since $\frac{e_1}{|\vec{e}|} = \cos\alpha$, $\frac{e_2}{|\vec{e}|} = \cos\beta$, $\frac{e_3}{|\vec{e}|} = \cos\gamma$

we have the law of cosines:

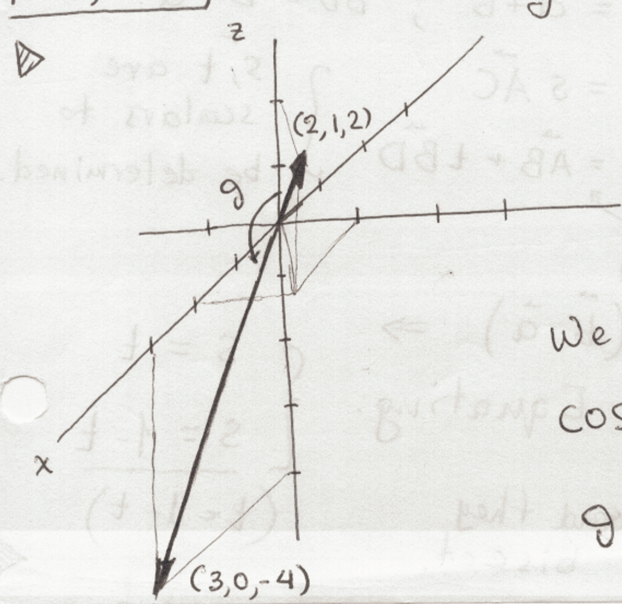
$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.}$$

Since $\cos\alpha = \cos\beta = \frac{1}{2}$, we have $\cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$

$$\Rightarrow \cos\gamma = \pm\frac{1}{\sqrt{2}} \Rightarrow \gamma = \begin{cases} \pi/4 \\ 3\pi/4 \end{cases}$$

$\therefore$ two vectors: $\vec{e} = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$; $\vec{e}' = \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}$

p.23, 1.7.1 Find the angle between $2\vec{i}+\vec{j}+2\vec{k}$ and $3\vec{i}-4\vec{k}$.



Let $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$

$\vec{b} = 3\vec{i} - 4\vec{k}$

Then $|a| = \sqrt{2^2+1^2+2^2} = 3$

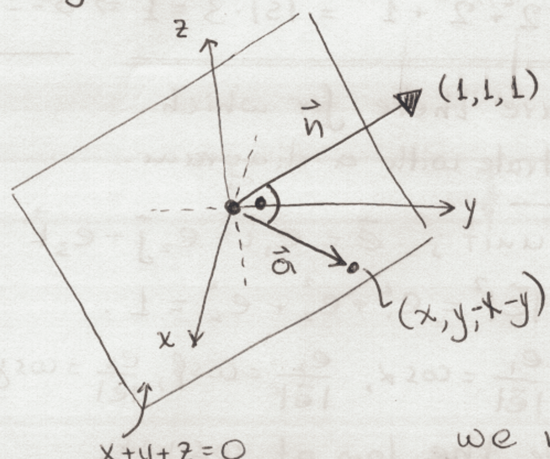
$|b| = \sqrt{3^2+0^2+4^2} = 5$

We have $\vec{a}\cdot\vec{b} = 2\cdot 3 + 1\cdot 0 + 2\cdot(-4) = -2$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{-2}{3\cdot 5} = -\frac{2}{15}$$

$$\theta = \cos^{-1}\left(-\frac{2}{15}\right) =$$

p.23, 1.7.5 Show that $\vec{i}+\vec{j}+\vec{k}$ is perpendicular to the plane $x+y+z=0$.



Let $\vec{n}=\vec{i}+\vec{j}+\vec{k}$

▷ Consider an arbitrary point on the plane

$$x+y+z=0. \quad (*)$$

This means that this point has coordinates (x,y,z) which satisfy $(*)$.

Then, if we let x, y be anything we must have $z=-x-y$.

So, any vector on the plane has the form

$$\vec{a}=x\vec{i}+y\vec{j}+(-x-y)\vec{k}$$

and $\vec{a}\bullet\vec{n} = x\cdot 1 + y\cdot 1 + (-x-y)\cdot 1 = x+y-x-y = 0$ ◁

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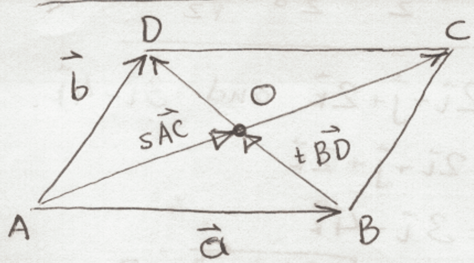
p.24, 1.7.9



"Show that the diagonals of a parallelogram bisect each other"

◁ Let $\left.\begin{aligned}\vec{AB}=\vec{DC}=\vec{a}\\ \vec{AD}=\vec{BC}=\vec{b}\end{aligned}\right\}$ <u>not collinear</u>

The diagonals are

$$\vec{AC}=\vec{a}+\vec{b} \quad ; \quad \vec{BD}=\vec{b}-\vec{a}$$

$\left.\begin{aligned}&\text{Let } \vec{AO}=s\vec{AC}\\ &\text{or } \vec{AO}=\vec{AB}+t\vec{BD}\end{aligned}\right\}$ s, t are scalars to be determined.

Setting these equal:

$$s\vec{AC}=s(\vec{a}+\vec{b}) = \vec{a}+t(\vec{b}-\vec{a}) \Rightarrow$$

$$\Rightarrow s\vec{a}+s\vec{b}=(1-t)\vec{a}+t\vec{b}. \text{ Equating: } \left\{\begin{aligned}s&=t\\ s&=1-t\end{aligned}\right. \quad (t=1-t)$$

$$\Rightarrow s=t=\frac{1}{2}$$ and they bisect. ◁