Math. 311
Set 14

4.2, p. 196
(2, 5, 6)

4.2.1 (Is region a domain? If yes is it simply connected)

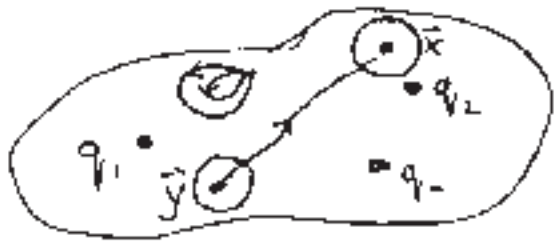
Definition: A point set that is

(i) open (each point is an interior point, i.e. $\vec{x} \in D$ means there is an open ball about $\vec{x}$, contained entirely in D)

(ii) connected: $\vec{x}, \vec{y} \in D \Rightarrow$ there is a curve of points in D that connects $\vec{x}$ to $\vec{y}$.

(D is simply connected: any closed path in D can be shrunk down to a point without leaving D)

4.2.2 The region of definition of an electric field due to n point charges is a simply connected domain: it is all of $\mathbb{R}^3$ excluding only the points where charges are situated. Then, any point is interior to D, and any loop can be shrunk to a point, avoiding the point charges



(that is possible in 3d but not in 2 dimensions!)

4.2.5 The region D consisting of all points (x, y, z) such that $x^2+y^2+z^2 > 4$

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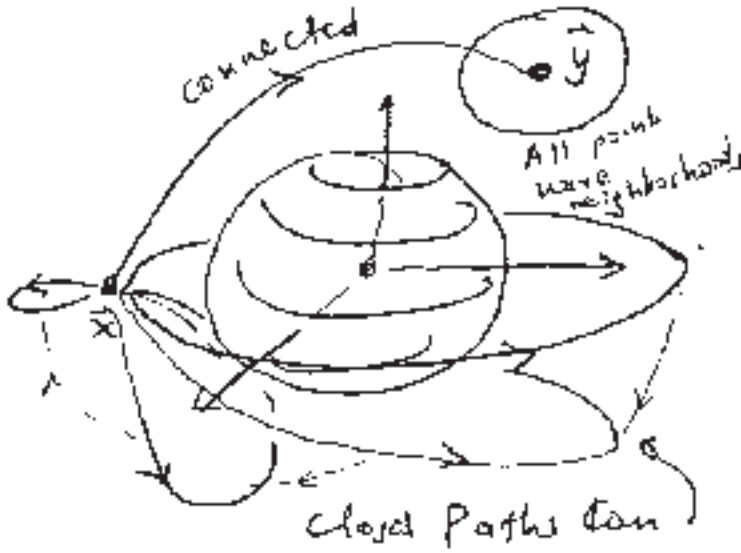
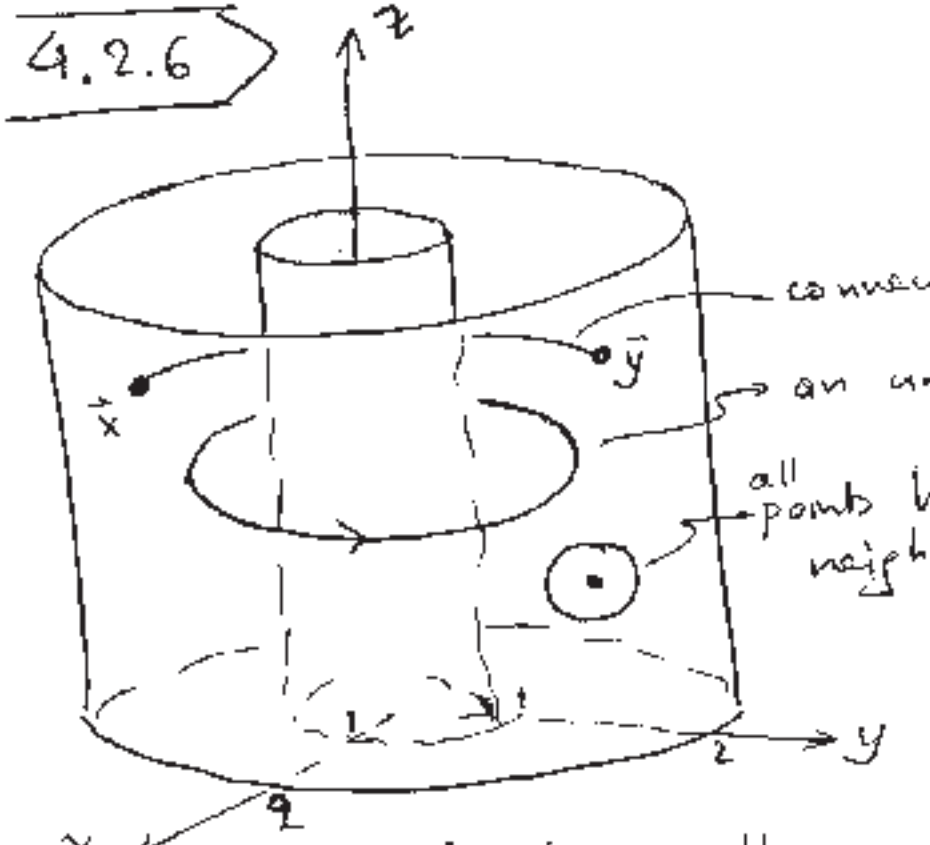
This is the exterior of a sphere of radius 4, center at the origin.

This is a simply connected domain



4.2.6



$1 < x^2+y^2 < 4$ is the annular region between two concentric cylinders. It is open and connected, but not simply connected

↳ It is a domain