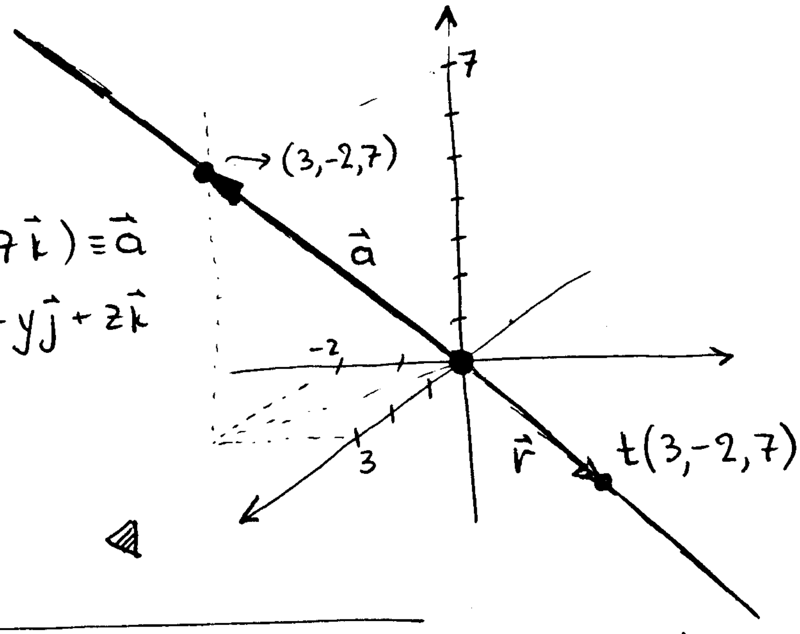
Math. 311
Spring '99
Homework 2

p.29, 1.8(1,9)
p.34, 1.9(13)
p.38, 1.10(2,13

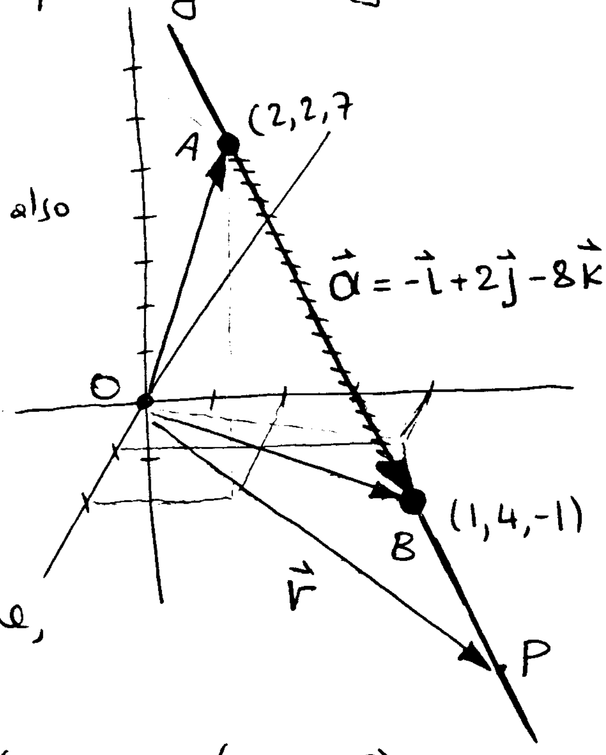
p.29, 1.8.1 > Find the parametric equations of the line passing through the origin parallel to $3\vec{i}-2\vec{j}+7\vec{k}$.




The line must contain any multiple of the vector $(3\vec{i}-2\vec{j}+7\vec{k}) \equiv \vec{a}$ i.e. its arbitrary point $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ must have the form

$$\vec{r}=t\vec{a} \Rightarrow \begin{cases} x = 3t \\ y = -2t \\ z = 7t \end{cases}$$

p.29, 1.8.9 > Find equations of the line passing through the points $(1,4,-1)$, and $(2,2,7)$.




Since the desired line contains the two points, A (1,4,-1), B (2,2,7), it also contains the vector

$$\vec{a}=\overrightarrow{AB}=(2-1, 4-2, -1-7)=$$
$$=-\vec{i}+2\vec{j}-8\vec{k}$$

Then, if $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ is an arbitrary vector to a point on the line, it must be of the form

$$\vec{r}=\vec{OA}+\vec{AP}=\vec{OA}+t\cdot\vec{a}=(2,2,7)+t(-1,2,-8)$$

i.e. $$\boxed{\begin{aligned} x &= -t+2 \\ y &= 2t+2 \\ z &= -8t+7 \end{aligned}}$$

$\boxed{2(2/4)}$

p.34, 1.9.13 Determine $s$ and $t$ so that $\vec{C}-s\vec{A}-t\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ given that

$$\begin{cases} \vec{A} = \vec{i}+\vec{j}+2\vec{k} \\ \vec{B} = 2\vec{i}-\vec{j}+\vec{k} \\ \vec{C} = 2\vec{i}-\vec{j}+4\vec{k} \end{cases}$$

let $\vec{D}=\vec{C}-s\vec{A}-t\vec{B}$

We have:
$$\begin{cases} \vec{A}\cdot\vec{A} = 1+1+4=6 \\ \vec{A}\cdot\vec{B} = 1\cdot2+1\cdot(-1)+2\cdot1 = 3 \\ \vec{A}\cdot\vec{C} = 1\cdot2+1\cdot(-1)+2\cdot4 = 9 \\ \vec{B}\cdot\vec{B} = 2\cdot2+(-1)(-1)+1\cdot1 = 6 \\ \vec{B}\cdot\vec{C} = 2\cdot2+(-1)(-1)+1\cdot4 = 9 \end{cases}$$

Now $\vec{D}\cdot\vec{A} = (\vec{C}-s\vec{A}-t\vec{B})\cdot\vec{A} = \vec{C}\cdot\vec{A} - s\vec{A}\cdot\vec{A} - t\vec{B}\cdot\vec{A} = 0$

$\Rightarrow 9 - s\cdot6 - t\cdot3 = 0$

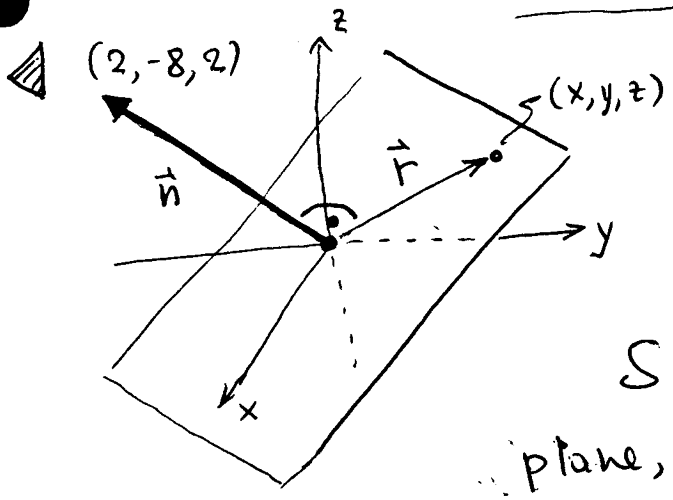
$\vec{D}\cdot\vec{B} = (\vec{C}-s\vec{A}-t\vec{B})\cdot\vec{B} = \vec{C}\cdot\vec{B} - s\vec{A}\cdot\vec{B} - t\vec{B}\cdot\vec{B} = 0$

$\Rightarrow 9 - s\cdot3 - t\cdot6 = 0$

$$\Rightarrow \left.\begin{array}{l} 6s+3t = 9 \\ 3s+6t = 9 \end{array}\right\} \quad s=t=1 \text{ (by inspection)}$$

$$\left(\text{or: } \left.\begin{array}{l} 2s+t = 3 \\ s+2t = 3 \end{array}\right\} \Rightarrow \begin{array}{l} t = 3-2s \\ s+2(3-2s) = 3 \Rightarrow -3s+6 = 3 \Rightarrow s=3 \text{ etc} \end{array}\right).$$

p.38, 1.10.2 > Find an equation of the plane through the origin perpendicular to $2\vec{i} - 8\vec{j} + 2\vec{k}$.



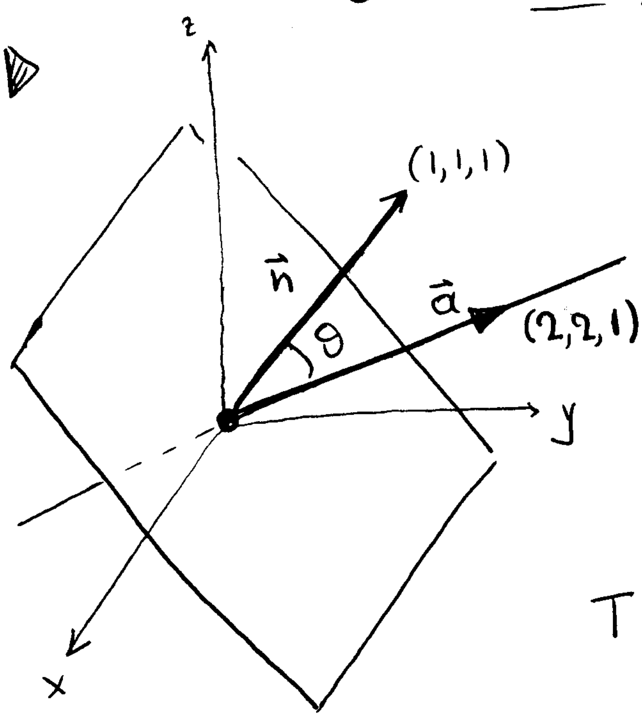

Let $\vec{r}$ be an arbitrary vector from the origin to a point of the plane $(x,y,z)$:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}.$$

Since the vector $\vec{n}$ is normal to the plane, we demand:

$$\vec{n} \cdot \vec{r} = 0 \Rightarrow \boxed{2x - 8y + 2z = 0}$$

p.38, 1.10.13 > By vector methods find the angle between the line $x = y = 2z$ and the plane $x + y + z = 0$.




The vector $\vec{n} = \vec{i} + \vec{j} + \vec{k}$ is normal to the plane (see problem p. 23, 1.7.5)

For the line, let

$$\left.\begin{aligned} x &= 2t \\ y &= 2t \\ z &= t \end{aligned}\right\} \text{ i.e. } \vec{r}(t) = \vec{a}t$$

with $\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}$

Then $\cos\vartheta = \dfrac{\vec{n} \cdot \vec{a}}{|n||a|} = \dfrac{5}{3\sqrt{3}}$

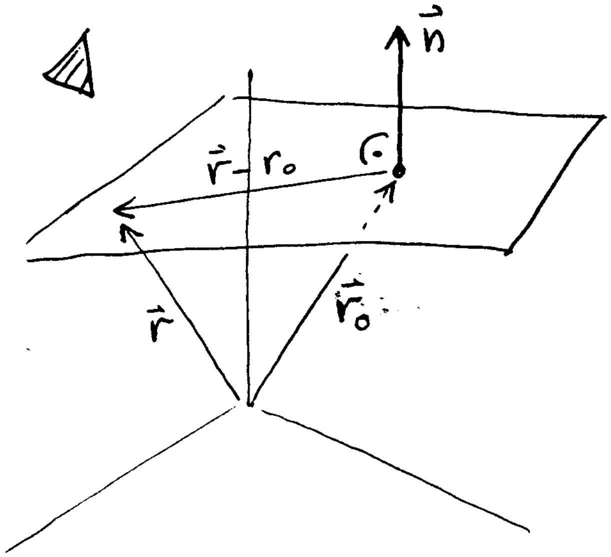
with $\vec{n} \cdot \vec{a} = 2 + 2 + 1 = 5$

$|n| = \sqrt{3}$

$|a| = 3$

$$\boxed{\vartheta = \cos^{-1}\left(\frac{5}{3\sqrt{3}}\right)}$$

P.39, 1.10.4 Find a plane crossing through $(1,3,3)$, parallel to the plane $3x+y-z=8$.




Let: $\vec{n} = a\vec{i}+b\vec{j}+c\vec{k}$ be normal to plane.

$\vec{r}_0 = x_0\vec{i}+y_0\vec{j}+z_0\vec{k}$ a given point on the plane

$\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ arbitrary point on plane.

Then $\vec{n} \perp \vec{r}-\vec{r}_0$ i.e.

$$\vec{n}\cdot(\vec{r}-\vec{r}_0)=0$$

$$\Rightarrow (a\vec{i}+b\vec{j}+c\vec{k})\cdot((x-x_0)\vec{i}+(y-y_0)\vec{j}+(z-z_0)\vec{k})=0$$

$$\Rightarrow a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

is equation of plane.

Here $\vec{r}_0 = \vec{i}+3\vec{j}+3\vec{k}$ while we are given that plane is parallel to $3x+y-z=8$.

Since parallel planes have the same normal, we deduce that $\vec{n} = 3\vec{i}+\vec{j}-\vec{k}$

(compare equations: the coefficients of $x, y, z$ are $a, b, c$.).

Then

$$3(x-1)+(y-3)-(z-3)=0 \Rightarrow$$

$$\boxed{3x+y-z=3}$$