Math. 311
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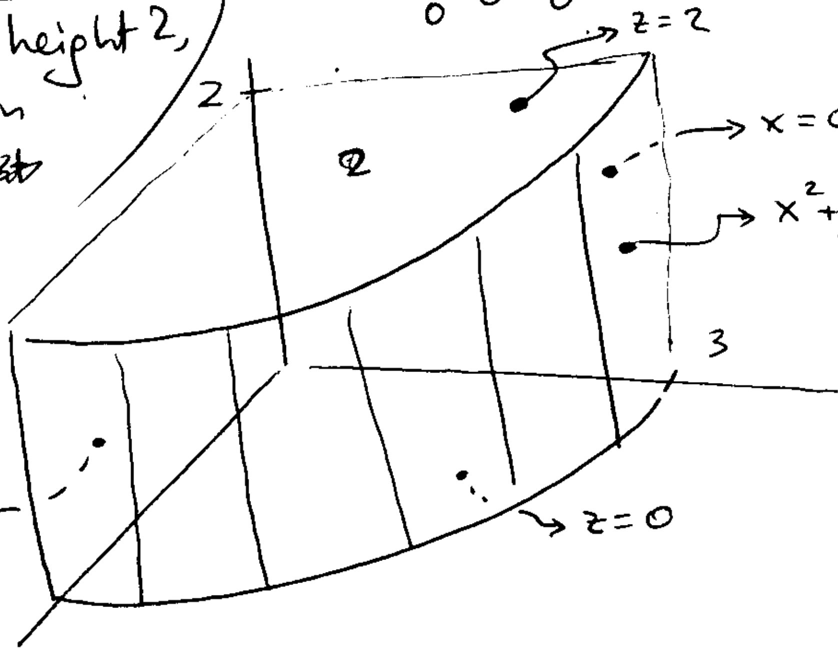
Sec. 4.8 (3,5,6)

p.256, 4.8.3 ⟩ Sketch the region whose volume is represented by the triple integral:

$$\int_0^2\int_0^3\int_0^{\sqrt{9-y^2}} dx\,dy\,dz = \frac{9\pi \cdot 2}{4} = \frac{9\pi}{2}$$

(Cylinder of height 2, radius 3, in 1st octant)

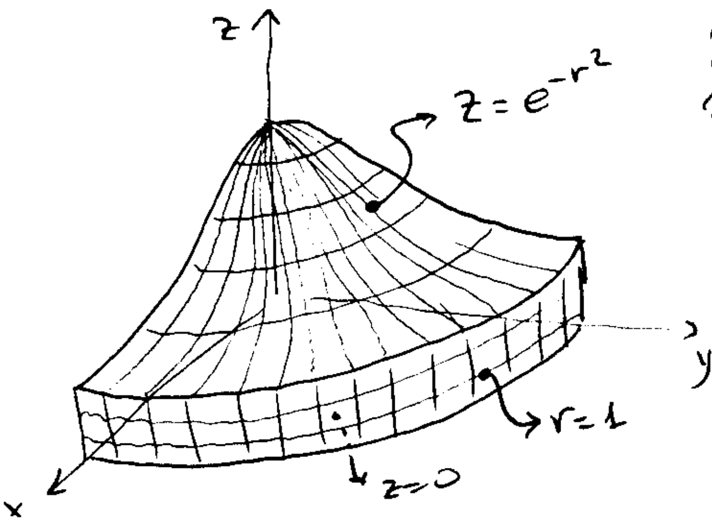


p.256, 4.8.5 ⟩ Let V be a domain with volume $\mathcal{V}$. Let $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$. (a) What is $\iiint_V \nabla\cdot\vec{F}\,dv$? (b) On the basis of the answer to exercise 4, what do you conjecture is the value of $\iint_S F\cdot dS$ of the flux of $\vec{F}$ over ∂V?

(a) $\nabla\cdot F = 3$; $\displaystyle\iiint_V \nabla\cdot F\,dv = 3\iiint_V dV = 3\mathcal{V}$

(b) Since $\displaystyle\oiint_{\partial V} F\cdot dS = \iiint \nabla\cdot F\,dV \Rightarrow \boxed{\oiint_{\partial V} \vec{F}\cdot d\vec{S} = 3\mathcal{V}}$

4.8.6 Find the volume of the region bounded by the surface $z = e^{-(x^2+y^2)}$, the cylinder $x^2+y^2=1$ and the plane $z=0$.



(only 1st octant shown)

Use cylindrical coords:

$$dV = r\,dr\,d\theta\,dz$$

$$V = \int_{\theta=0}^{2\pi} d\theta \int_{r=0}^{1} dr\, r \int_{z=0}^{e^{-r^2}} dz =$$

$$= 2\pi \cdot \int_0^1 dr\, r\, e^{-r^2}$$

$$= -\pi e^{-r^2}\Big|_0^1$$

$$\Rightarrow \boxed{V = \pi\left(1-e^{-1}\right)}$$