Solutions, 311-XXVI

April 28, 2003

Sec. 5.(5) Schaum's, p.132(40*,41,42*,52*,54*), Text,p.247,(16*)

1 Problem S6.40, p.132

Evaluate $\oint(x^2 - 2xy)sx + (x^2y + 3)dy$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$ (a) directly (b) by Green's theorem.
(*Ans.* 128/5)
Solution:

2 Problem S6.42

Evaluate $\oint (3x^2 + 2y)dx - (x + 3\cos y)dy$ around the parallelogram having vertices at $(0, 0)$, $(2, 0)$, $(3, 1)$ and $(1, 1)$.
($Ans.$ -6)
Solution:

3 Problem S6.52, p.133

Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ with $\mathbf{F} = 2xy\mathbf{i} + yz^2\mathbf{j} + xz\mathbf{k}$ and S is:

1. The parallelepiped $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$, $z = 3$.
2. The surface of the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $y = 3$ and $x + 2z = 6$.

(*Ans.* (a) 30, (b) 351/2)
Solution:

4 Problem S6.54, p.133

Evaluate $\int\int_S \mathbf{r} \cdot \mathbf{n} dS$ where

1. S is the sphere of radius 2 with center at $(0, 0, 0)$.

2. S is the surface of the cube bounded by the planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$.

3. S is the surface bounded by the paraboloid $z = 4 - (x^2 + y^2)$ and the xy plane.

(*Ans.* (a) 32π, (b) 24, (c) 24π)
Solution:

5 Problem 4.7.16, p.247

A torus (donut) has major radius A and minor radius a (see figure). Derive the parametrization $\mathbf{R}(u, v)$ in terms of the toroidal angle u and the poloidal angle v, where

$$\begin{aligned} x &= A\cos u + a\cos u \cos v \ , \\ y &= A\sin u + a\sin u \cos v \ , \\ z &= a\sin v \ . \end{aligned}$$

Show that the area of the torus is $4\pi^2 Aa$.
Solution: