Solutions, 311-XXVII

April 28, 2003

Sec. 5.2 (1*,2*,8a*), S6.74*, p.134

1 Problem 5.2.1, p.284

Evaluate

$$\int\int_S \left[\frac{1}{R}\nabla\phi - \phi\nabla\left(\frac{1}{R}\right)\right]\cdot d\mathbf{S}$$

over the surface of the sphere $(x-3)^2+y^2+z^2=25$, where $\phi = xyz+5$.
Solution:

2 Problem 5.2.2, p.284

Evaluate

$$\int\int_S \left[\phi\nabla\left(\frac{1}{R}\right) - \frac{1}{R}\nabla\phi\right]\cdot d\mathbf{S}$$

1. over the surface of the ellipsoid $x^2/9 + y^2/16 + z^2/25 = 1$, where $\phi = x^2 + y^2 - 2z^2 + 4$.

2. over the surface of the cylindrical pillbox bounded by $x^2 + y^2 = 25$ and $z = \pm 10$, where $\phi = x^2 - z^2 + 5$.

Solution:

3 Problem 5.2.8a, p.285

Let the domain D be bounded by the surface S as in the divergence theorem, and assume that all fields satisfy the appropriate differentiability conditions. Prove the identity:

$$\int\int\int_D \nabla\phi \cdot \nabla \times \mathbf{F} dV = \int\int_S \mathbf{F} \times \nabla\phi \cdot d\mathbf{S} \ .$$

Solution:

4 Problem S6.74, p.134

If a region V bounded by a surface S has a continuous charge (or mass) distribution of density ρ, the potential $\phi(P)$ at a point P is defined by

$$\phi = \int\int\int_V \frac{\rho dV}{r} .$$

Deduce the following under suitable assumptions:

1. $\int\int_S \mathbf{E} \cdot d\mathbf{S} = 4\pi \int\int\int_V \rho dV$, where $\mathbf{E} = -\nabla\phi$.
2. $\nabla^2\phi = -4\pi\rho$ (Poisson's equation) at all points P where charges exist, and $\nabla^2\phi = 0$ (Laplace's equation) where no charges exist.

Solution: