p 284, #1) By Green's formula III, we only need to evaluate

$$4\pi\, \phi(0,0,0) + \iiint_V \nabla^2\phi / R \, dV,$$

where V is the volume bounded by S. Since $\nabla^2 \phi = 0$ and $\phi(0,0,0) = 5$, we get

$$\iint_S \left[\frac{1}{R} \nabla\phi - \phi \nabla\left(\frac{1}{R}\right) \right] \cdot \hat{n} \, dS = 20\pi .$$

p 143, #10) a. On p. 52 of Schey, the continuity eq is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

where ρ is the density and v is the velocity. Since ρ is constant, $\partial \rho / \partial t = 0$ and $\nabla \cdot (\rho v) = \rho \nabla \cdot v$, s.t. $\nabla \cdot v = 0$.

b. Since $\nabla \times v = 0$, there is a function Φ s.t. $v = \nabla \Phi$. Since the fluid is incompressible, $\nabla \cdot v = 0$. Thus,

$$\nabla \cdot \nabla \Phi = \nabla^2 \Phi = 0.$$

p 143, #13) a. Since $\nabla \cdot E = \rho / \epsilon_0$, we get $\rho = \epsilon_0 \nabla \cdot E = 3g\, \epsilon_0$.

b. Since $E = -\nabla \phi$, we see that $\phi = -\frac{g}{2}(x^2 + y^2 + z^2)$

c. It is trivial to check that

$$\nabla^2 \phi = -3g \quad (= -\rho / \epsilon_0).$$