311-TEST I

Name:____

September 20, 2006

(1 − 25pts) Consider the helix $x = 5\sin 4t$, $y = 5\cos 4t$, $z = 10t$. In (a-d) find as functions of time:

1. The speed
2. The tangential and normal components of acceleration
3. The unit tangent vector $\mathbf{T}$.
4. The curvature of the curve
   and, in addition:
5. Find the arc length from $t = 0$ to $t = \pi/2$.

(2 − 13pts) Find an equation of the line passing through the points $(1, 4, -1)$ and $(2, 2, 7)$ in both parametric and non-parametric forms.

(3 – 15pts) Consider the scalar field $f(x, y, z) = x^2 + 2y^2 + 3z^2$.

1. Find the magnitude of the greatest rate of change at the point $(1, 3, -2)$ and
2. the unit vector in the direction of the greatest rate of change at the same point, $(1, 3, -2)$.
3. Find the directional derivative at $(1, 3, -2)$ in the direction of the vector $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

(4 – 20 pts) Consider the triangle ABC in the figure (1), where the lines AL and BS bisect the sides BC and AC respectively. Show that $\vec{OS} = \frac{1}{2}\vec{BO}$ or, equivalently, that $\vec{OS} = \frac{1}{3}\vec{BS}$ and $\vec{BO} = \frac{2}{3}\vec{BS}$.
(HINT: write $\vec{BO} = t\vec{BS}$ and also $\vec{BO} = \vec{BL} + r\vec{LO}$. Then express $\vec{BS}$, $\vec{BL}$ and $\vec{LO}$ in terms of the two non-colinear vectors $\vec{AC}$ and $\vec{BC}$ and determine t and r.)

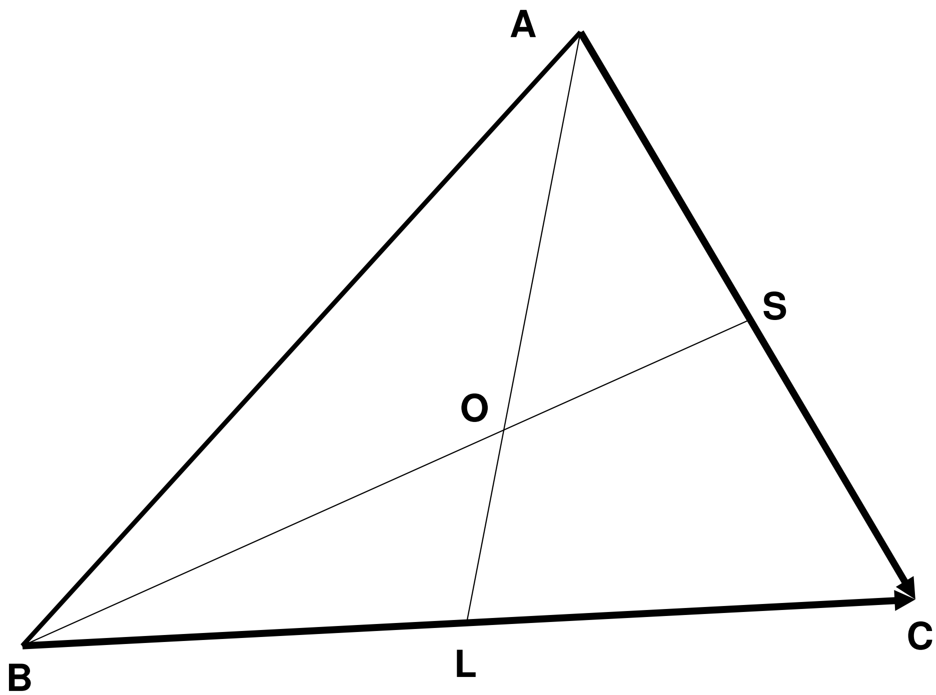

Figure 1: The triangle for problem 5

(5 – 15pts) Find an equation of the plane tangent to the surface

$$z - x^2 - y^2 = 0 \text{ at } (2, 3, 13) .$$

(6 − 12 pts) Consider the vector field $\mathbf{F} = e^{xy}\mathbf{i} + \sin xy\mathbf{j} + \cos^2 xz\mathbf{k}$. Find

1. $\nabla \cdot \mathbf{F}$.

2. $\nabla \times \mathbf{F}$.