311-TEST II

Name:____

September 20, 2006

INSTRUCTIONS:
ALL PROBLEMS ARE WEIGHTED EQUALLY!
DO ALL FOUR PROBLEMS!
One full page of notes is allowed.

Problem	grade
1	
2	
3	
4	
Total	

(1 – 25pts) Given that $\phi(x, y, z)$ is a differentiable scalar field, $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $R = |\mathbf{R}|$ and $\mathbf{A}$ is a constant vector field, compute (using appropriate vector identities, if you need to):

1. $\nabla \cdot (\mathbf{A} \times \mathbf{R})$
2. $\nabla (\mathbf{A} \cdot \mathbf{R})$
3. $\nabla \cdot \left(\dfrac{\mathbf{A} \times \mathbf{R}}{R} \right)$

   and, in addition:
4. Prove (give all details!) the vector identity:

$$\nabla \times \nabla\phi = 0 \ .$$

(2 − 25) Consider the vector field $\mathbf{F} = 2xy\mathbf{i} + (x^2 + z)\mathbf{j} + y\mathbf{k}$

1. Use an appropriate parametrization to compute the line integral $I = \int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the straight line from the point $(0, 0, 0)$ to the point $(1, 1, 1)$.

2. Show that $\mathbf{F}$ is irrotational and compute a scalar potential. Here you must carry out the computation explicitly, just guessing won't do!

3. Compute (by any method applicable) $I = \int_C \mathbf{F} \cdot d\mathbf{R}$ where now C is the curve $x = \sin\theta$, $y = \sin^2\theta$, $z = 2\theta/\pi$ with $0 \leq \theta \leq \pi/2$.

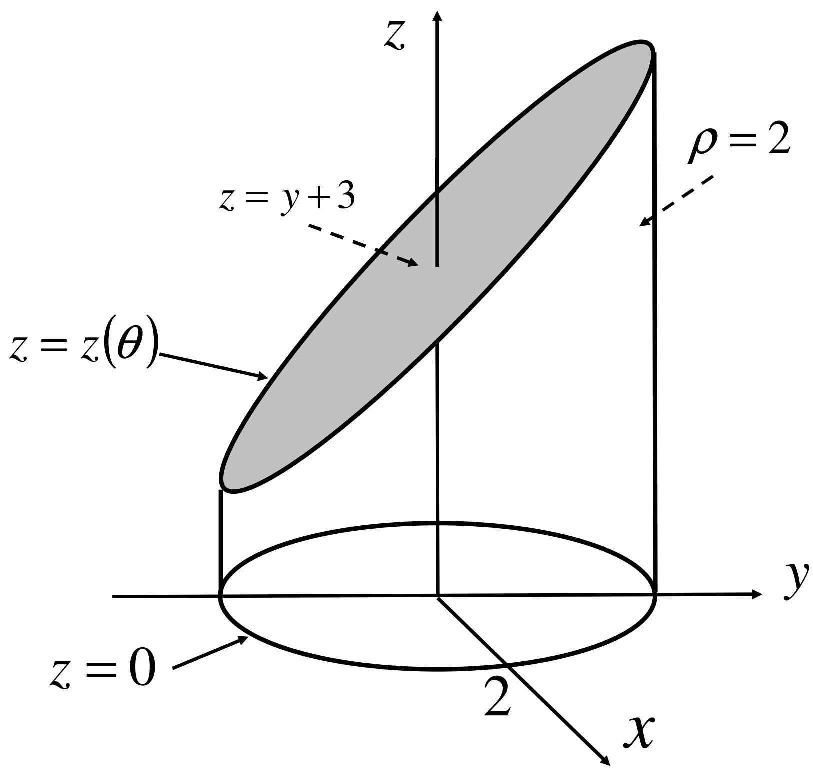

Figure 1: The cylinder for problem 3: the integration is to be carried out over the lateral surface

(3 – 25pts) Evaluate $\int\int_S z^2 dS$, where S is the part of the lateral surface of the right circular cylinder $x^2 + y^2 = 4$ between the planes $z = 0$ and $z = y + 3$.

(4 − 25 pts) Consider the triangle with vertices $(-1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 1)$. Compute the surface integral $\Phi = \int\int \mathbf{F} \cdot \mathbf{n} dS$ over the surface of the triangle, where $\mathbf{n}$ is the unit normal to the surface chosen so that its projection on the z-axis points in the positive z-direction (i.e. $\mathbf{n} \cdot \mathbf{k} > 0$) and $\mathbf{F}$ is the vector field $\mathbf{F} = xy\mathbf{i} - zx\mathbf{j} + y\mathbf{k}$.