311-TEST III

Name:____

May 2, 2005

INSTRUCTIONS:
ALL PROBLEMS ARE WEIGHTED EQUALLY!
DO ALL FOUR PROBLEMS!
One full page of notes is allowed.

Problem	grade
1	
2	
3	
4	
Total	

(1 – 25 pts)

1. (10 pts) What is the flux output per unit volume from an ellipsoid of volume V if $\mathbf{F} = 3x\mathbf{i} + 2y\mathbf{j} - z\mathbf{k}$?

2. (10 pts) Show that

$$\int\int_S \nabla\phi \times \nabla\psi \cdot d\mathbf{S} = \oint_C \phi\nabla\psi \cdot d\mathbf{R} \ .$$

3. (5 pts) Let $\mathbf{B} = \nabla \times \mathbf{A}$ where $\mathbf{A}$ is some continuously differentiable vector field. If C is a simple closed loop and S is a surface bounded by C, show that

$$\oint_C \mathbf{A} \cdot d\mathbf{R} = \int\int_S \mathbf{B} \cdot d\mathbf{S} \ ,$$

(2 − 25pts) Use Stokes' theorem to evaluate

$$I := \oint_C \left[x \sin y \mathbf{i} - y \sin x \mathbf{j} + (x+y) z^2 \mathbf{k} \right] \cdot d\mathbf{R}$$

along the path consisting of straight line segments successively joining the points $P_0 = (0,0,0)$ to $P_1 = (\pi/2, 0, 0)$ to $P_2 = (\pi/2, 0, 1)$ to $P_3 = (0,0,1)$ $P_4 = (0, \pi/2, 1)$ to $P_5 = (0, \pi/2, 0)$ and back to P_0.
(**NOTE:** you are to compute this integral by converting to appropriate surface integral(s); do not evaluate the line integral directly!)

(3 – 25 pts) Find $I := \oint_C \mathbf{F} \cdot d\mathbf{R}$ where C is the ellipse of intersection of the cylinder $x^2+y^2 = 1$ with the plane $z = x$ and $\mathbf{F} = (x+y)\mathbf{i}+y\mathbf{j}+(x+y+z)\mathbf{k}$:

(a – 12 pts) By direct evaluation of the line integral.

(b – 13 pts) By applying Stoke's theorem to convert to a surface integral over an appropriate surface (which you must compute!).

(4 − 25pts) Find the volume of the region bounded by the surface

$$z = \cos\left(x^2 + y^2\right) ,$$

the cylinder

$$x^2 + y^2 = \frac{\pi}{4} ,$$

and the $x - y$ plane.