311-TEST IV (final)

Name:____

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INSTRUCTIONS:
ALL PROBLEMS ARE WEIGHTED EQUALLY!
DO ALL FOUR PROBLEMS!
One full page of notes is allowed.

Problem	grade
1	
2	
3	
4	
Total	

1. **(25 pts)**

   (a) (13 pts) Find the curvature and torsion for the helix

$$x = \cos t \ , \ \ y = \sin t \ , \ \ z = t \ .$$

(b) (12 pts) Given: f is a scalar field, is a vector field (both are assumed sufficiently differentiable). **EXPLAIN** which of the following is a vector field, scalar field, meaningless or ambiguous (i.e. has a value that depends on how parentheses are placed).

i. $\mathbf{F} \times \nabla \times \nabla f$

ii. $\nabla \times f$

iii. $\nabla \times \nabla^2 \mathbf{F}$

iv. $\nabla \cdot (\nabla^2 f)$

v. $\nabla (\nabla^2 f)$

vi. $\nabla \cdot \nabla \times \mathbf{F}$

2. **(25pts)**

(a) (12 pts) Are the following fields conservative? If yes, find potential:

i. (7 pts)

$$\mathbf{F} = 3yx^2 e^{x^3 y}\mathbf{i} + x^3 e^{x^3 y}\mathbf{j} + z\mathbf{k}\ .$$

ii. (6 pts)

$$\mathbf{F} = x\mathbf{i} - y\mathbf{j}\ .$$

(b) (12 pts) Verify that the following field is both irrotational and solenoidal and compute a scalar and a vector potential:

$$\mathbf{F} = e^x \sin y \mathbf{i} + e^x \cos y \mathbf{j} \ .$$

3. **(25pts)** Given the vector field

$$\mathbf{F} = \left(x^2 - y^2\right)\mathbf{i} + 2xy\mathbf{j} \ .$$

Find $\oint_C \mathbf{F} \cdot d\mathbf{R}$ around the square with vertices $(0,0), (1,0), (1,1), (0,1)$ (in that order).

(a) (12 pts) By direct computation

(b) (13 pts) By using Stoke's theorem and evaluating the resulting surface integral.

4. **(25 pts)** Compute the flux of the vector field

$$\mathbf{F} = ze^y\mathbf{i} - xz\cos z\mathbf{j} + (z+1)\mathbf{k}$$

over the hemispherical shell $x^2 + y^2 + z^2 = 1$, $z \geq 0$, open at the bottom. (Hint: you can take advantage of the divergence theorem, to reduce this computation to that of computing an easy volume integral and an easy surface integral!)