312 '10-EXAM 1

October 20, 2010

You may use one page of notes. All problems are weighted equally.

Problem	grade
1	
2	
3	
4	
Total	

1. Solve the PDE

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = e^{y-2x} \ , \ \ u(0, y) = 0 \ .$$

2. (a) Classify the following PDE's as elliptic, hyperbolic or parabolic.

$$u_{xx} + u_{xt} + 2u_{tt} = 0 \tag{1}$$
$$u_{xx} + 2u_{xt} + u_{tt} = 0 \tag{2}$$
$$u_{xx} + 3u_{xt} + 2u_{tt} = 0 \tag{3}$$

(b) If any of the above PDE is hyperbolic, compute the solution to the initial value problem

$$u(x,0) = \phi(x) \ , \ u_t(x,0) = \psi(x) \ , \ -\infty < x < \infty \ .$$

3. (a) Use separation of variables to solve the initial-boundary value problem:

$$u_t = ku_{xx} , \quad u_x(0,t) = u_x(\pi,t) = 0 , \quad u(x,0) = x , \quad 0 \le x \le \pi .$$

(b) What is the limit of $u(x,t)$ as $t \to \infty$?

4. Consider the function

$$f(x) = \begin{cases} -1, & 0 < x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x < \pi \end{cases}.$$

(a) Find the Fourier sine and cosine series expansion for f over the interval $0 \le x \le \pi$.

(b) Give a sketch for the function represented by each over the interval $-2\pi \le x \le 2\pi$.