314 '07-QUIZ 2

Name: KEY

February 26, 2009

1 < $10pts$ >

Find all values of c for which the matrix $\mathbf{A}$ is singular, with

$$\mathrm{A} = \begin{pmatrix} 1-c & -2 & 2 \\ 0 & 2-c & 3 \\ 0 & 0 & 3-c \end{pmatrix}$$

Solution (1, 2, 3)

$$\mathrm{Det}\,A = (1-c)(2-c)(3-c) = 0$$

$$\Rightarrow \quad c = 1, 2, 3$$

2 < 10pts >

A top secret message was encoded into the string

$$-1, 12, -14, -4, 15, -15, 9, 18, -30, 5, 5, -35$$

using the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

and the correspondence:
space $= 0, A = 1, B = 2, \ldots, Z = 26, (. \; , \; ? \; ! \; : \; *) = (27, \ldots, 32)$. What was the message?

Solution

$$\begin{matrix} \\ -1 \\ 1 \end{matrix}\left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right)\begin{matrix} -1 \\ 1 \\ \\ \end{matrix}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right) \leftarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right)\begin{matrix} 1 \\ \\ \\ \end{matrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} -1 & -4 & 9 & 5 \\ 12 & 15 & 18 & 5 \\ -14 & -15 & -30 & -35 \end{pmatrix} = \begin{pmatrix} 12 & 15 & 18 & 5 \\ 15 & 19 & 21 & 30 \\ 2 & 0 & 12 & 30 \end{pmatrix}$$

$$\begin{pmatrix} L & O & R & E \\ O & S & U & ! \\ B & \_ & L & ! \end{pmatrix}$$

"LOBOS_RULE!!"

A	1	p	16
B	2	q	17
C	3	r	18
D	4	s	19
E	5	t	20
F	6	u	21
G	7	v	22
H	8	w	23
I	9	x	24
J	10	y	25
K	11	z	26
L	12		
M	13		
N	14		
O	15		

3 < 10*pts* >

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 0 & -2 & 2 \end{pmatrix}$$

1. Use Gauss elimination to find the determinant $det\mathbf{A}$
2. What is the determinant of the inverse, $det\mathbf{A}^{-1}$?

Solution

$$\begin{array}{c} \\ 1 \\ -1 \\ -1 \end{array}\begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & 0 & -2 & 2 \end{pmatrix} \rightarrow \begin{array}{c} \\ \\ -1 \\ -2 \end{array}\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & -2 & -1 & 1 \end{pmatrix} \rightarrow \begin{array}{c} \\ \\ \\ -1 \end{array}\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\det A = -2$$

$$\frac{1}{\det A} = \det(A^{-1}) = -\frac{1}{2}$$

$$\begin{array}{c} \\ 1 \\ -2 \end{array}\begin{pmatrix}1&-1&1\\-1&2&-1\\2&-1&3\end{pmatrix}\to\begin{pmatrix}1&-1&1\\0&1&0\\0&1&1\end{pmatrix}\to\begin{pmatrix}1&-1&1\\0&1&0\\0&0&1\end{pmatrix}$$

$\det A = 1$

4 $< 10pts >$

Use Cramer's rule to solve the system

$$\begin{array}{rcrcrcl} x_1 & - & x_2 & + & x_3 & = & 1 \\ -x_1 & + & 2x_2 & - & x_3 & = & 0 \\ 2x_1 & - & x_2 & + & 3x_3 & = & 0 \end{array}$$

Solution

$$A = \begin{pmatrix}1&-1&1\\-1&2&-1\\2&-1&3\end{pmatrix}, \quad \det A = 1\begin{vmatrix}2&-1\\-1&3\end{vmatrix} + 1\begin{vmatrix}-1&-1\\2&3\end{vmatrix} + 1\begin{vmatrix}-1&2\\2&-1\end{vmatrix} = 5 + (-1) + (-3) =$$

$$x_1 = \begin{vmatrix}1&-1&1\\0&2&-1\\0&-1&3\end{vmatrix} = \begin{vmatrix}2&-1\\-1&3\end{vmatrix} = 5$$

$$x_2 = \begin{vmatrix}1&1&1\\-1&0&-1\\2&0&3\end{vmatrix} = -\begin{vmatrix}-1&-1\\2&3\end{vmatrix} = 1$$

$$x_3 = \begin{vmatrix}1&-1&1\\-1&2&0\\2&-1&0\end{vmatrix} = \begin{vmatrix}-1&2\\2&-1\end{vmatrix} = -3$$

$$\left.\begin{array}{l} 5-1-3=1 \\ -5+2+3=0 \\ 10-1-9=0 \end{array}\right\}$$

$\cancel{\tfrac{7}{3} - \tfrac{1}{3} - 1 = 1}$

$\cancel{-\tfrac{4}{3} + \tfrac{2}{3} + 1 =}$