314, S'09-QUIZ 4

Name:____

April 9, 2009

1 $< 12pts >$

Let **B** and **C** be fixed $n \times n$ matrices. Determine whether the following are linear operators on $\mathbf{R}^{n\times n}$.

1. (4 pts) $L(A) = C^2A + AB$.
2. (4 pts) $L(A) = C^2A + B$.
3. (4 pts) $L(A) = C^2AB$.

Solution

2 $< 16pts >$

Given the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \in \mathbf{R}^3$

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} , \mathbf{u}_2 = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} , \mathbf{u}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

and the vectors $\mathbf{v}_1, \mathbf{v}_2 \in \mathbf{R}^2$,

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} , \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and let $\mathcal{L}$ be the linear transformation from $\mathbf{R}^3$ to $\mathbf{R}^2$ given by

$$(y_1, y_2)^T = \mathcal{L}(x_1, x_2, x_3) = (x_2 - x_1, x_3 - x_1)^T .$$

1. (5pts) Find a matrix $\mathbf{A}$ such that $\mathbf{y} = \mathcal{L}(\mathbf{x}) = \mathbf{A}\mathbf{x}$ for all inputs $\mathbf{x} \in \mathbf{R}^3$ written in the standard basis $E_3 = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and giving output $\mathbf{y} \in \mathbf{R}^2$ in terms of its components in the standard basis $E_2 = (\mathbf{e}_1, \mathbf{e}_2)$.

2. (8pts) Find a matrix $\mathbf{B}$ such that $\mathcal{L}(\mathbf{x}) = \mathbf{B}\mathbf{x} = \mathbf{y}$ for all input vectors $\mathbf{x} \in \mathbf{R}^3$ written in the basis $U = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ and giving output $\mathbf{y} \in \mathbf{R}^2$ in terms of its components in the basis $V = (\mathbf{v}_1, \mathbf{v}_2)$.

3. (3pts) Are the matrices $\mathbf{A}$ and $\mathbf{B}$ related? How?

Solution

3 $< 12pts >$

1. (2pts) Show that the following transformation from P^2 to P^3 is linear: $\mathcal{L}(p(x)) = p(1) + \int_0^x p(x)dx$.

2. (10pts) Write the above linear transformation in terms of a 2×3 matrix if we use the ordered basis $U = \{1-x, 1+x\}$ for P^2 and the standard ordered basis $E = \{1, x, x^2\}$ for P^3. That is, write $\mathcal{L}(\mathbf{p}) = \mathbf{q}$ as $\mathbf{A}\mathbf{p}_U = \mathbf{q}_E$ where $\mathbf{p}(x) = a_1(1-x) + a_2(1+x) \Rightarrow \mathbf{p}_U = (a_1, a_2)^T$; $\mathbf{q}(x) = b_1 + b_2x + b_3x^2 \Rightarrow \mathbf{q}_E = (b1, b_2, b_3)^T$ and

$$A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} .$$

Solution