

314 Spring '09-QUIZ 5

Name:-----

April 30, 2009

1 < 10pts >

Let $\mathbf{x} = (1, -1, 1)^T$, $\mathbf{y} = (-1, 1, -2)^T$. Compute $\|\mathbf{x} - \mathbf{y}\|_1$, $\|\mathbf{x} - \mathbf{y}\|_2$, $\|\mathbf{x} - \mathbf{y}\|_\infty$. Under which norm are the two vectors closest together? Under which norm are they farthest apart?

Solution

2 < 10pts >

Find the vector projection \mathbf{p} of \mathbf{x} onto \mathbf{y} and verify that \mathbf{p} and $\mathbf{x} - \mathbf{p}$ are orthogonal if $\mathbf{x} = (1, -1, 1)^T$, $\mathbf{y} = (-1, 1, -2)^T$.

Solution

3 < 20pts >

Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -2 \end{pmatrix}$$

1. Use the Gram-Schmidt process to construct the QR factorization.
2. Use the QR factorization found in part (1) to solve the least squares problem $\mathbf{A}\mathbf{x} = (1, -1, 0, 1)^T$.

Solution